Volume XVII. *November, 1903.* *Number 5.*

THE

PHYSICAL REVIEW.

THE LUMINOUS EFFICIENCY OF VACUUM-TUBE RADIATION.

By E. R. Drew.

THE radiation which is produced when an electric current is passed through a rarefied gas has a special interest on account of the comparatively small increase in temperature which accompanies it. The early investigators seem to have assumed that the temperature in the vacuum-tube discharge is of the same order as that in luminous flames. Hittorf among others doubted this, and adduced several considerations[1] which tended to show that it must be much lower. E. Wiedemann[2] immersed the capillary portion of a vacuum-tube in a calorimeter, and measured the heat developed by each discharge of an induction coil through it. Apparently assuming that the heat developed first raised the temperature of the gas before any appreciable conduction took place through the glass walls of the tube, he calculated the rise in temperature from the specific heat of the gas. His results are of the order of 80,000° for the capillary, which is doubtless too high; but he concluded that in the wider part of the tube there might be considerable light radiation with a temperature below 100°.

The portion of a vacuum tube immediately surrounding the anode, and that surrounding and, at lower pressures, extending some distance from the cathode, does not lend itself readily to such investigations, because of the lack of uniformity in the electrical conditions. For this reason the so-called "positive column," which extends

[1] Wied. Ann., 7, 575, 1879.
[2] Wied. Ann., 6, 298, 1879

from just in front of the anode some distance toward the cathode, where these conditions approach much more nearly to uniformity, has been the usual subject of study. Warburg[1] calculated the temperature in this region for several cases, on the assumption that the electrical energy which disappears is all converted into heat; this being measured by the product of potential gradient and current strength. The results thus obtained agree quite well with the values later found by Wood,[2] who measured the temperature directly by means of a bolometer wire inserted in the tube across the path of the discharge. The temperatures thus found are often only a few degrees above that of the surrounding air.

K. Ångström was one of those who recognized the fact that the radiation from a vacuum-tube must be of a different character in some respects from that which comes from incandescent gases and solids. One of the characteristics of the latter class, which radiate on account of their high temperature, is that the radiation covers the whole range of frequencies up to the highest which is present, this latter, and also the proportion of high-frequency radiation to the total, increasing in general with the temperature. Ångström[3] measured the total radiation from a vacuum-tube, by means of a bolometer, and also the portion which is transmitted by a plate of alum, under different conditions. The ratio of this portion to the total may for convenience be called the luminous efficiency, although it is well known that the radiation transmitted by the alum plate extends somewhat beyond the red end of the visible spectrum. He found that this ratio is quite different in different gases, while for the same gas it increases continuously as the pressure decreases, and at a given pressure is apparently independent of the current. The chief point of interest, however, is the fact that the efficiency is much higher than is the case with sources of light which radiate on account of their high temperature. In the case of nitrogen at a pressure of 0.15 mm. it reaches a value above ninety per cent.

Ferry[4] measured the changes in the luminous intensity of bright lines in the spectra of some gases, using a spectro-photometer, and

[1] Wied. Ann., 54, 265, 1895.
[2] Wied. Ann., 59, 238, 1896.
[3] Wied. Ann., 48, 493, 1893.
[4] PHYSICAL REVIEW, 7, 9, 1898.

found them to be in agreement with the corresponding results which Ångström had found for the total radiation.

These results make it highly probable, as Angström and others have pointed out, that when radiation is emitted by a gas, in one case by heating it, and in another case by sending a current through it, the mechanism which is brought into play must differ in some important respects in the two cases. It was in the hope of gaining some further knowledge in regard to this mechanism that the present investigation was begun, along the same general lines as that of Ångström. After giving an account of the experimental methods and results, an attempt will be made to account for the latter, so far as possible, in terms of the modern electron theory, which has proved so useful in its application to the varied phenomena which attend the passage of electricity through gases.

Experimental.

Radiometer. — A radiometer was used for measuring the radiation from the tube; partly because the magnetic disturbances about the physical laboratory make it troublesome to work with the sensitive galvanometer which would be required for a bolometer or thermopile, and partly because it has some inherent advantages over the latter. It is less trouble to construct and use at a high degree of sensitiveness, and its lack of mobility is no objection for the present purpose. The instrument as constructed happens to correspond quite accurately, in its essential features, with the one which E. F. Nichols[1] used in his determination of the radiation from stars and planets. The suspended system is larger, the mica vanes being 3 mm. in diameter instead of 2, and 7 mm. between centers. It weighs about 12 mg. The window through which radiation enters at *f*, Fig. 3, is of fluorite, very clear and about 1 mm. thick. The vanes are about 2 mm. behind this window, at *v*. A small glass window *g*, behind the vanes, through which they can be seen, is set at an angle, so that radiation entering through the window in front and passing the edge of the vane will not be directly reflected upon its rear face, but will fall instead upon the blackened interior walls of the case. When the necessary screens were put in front of the fluo-

[1] Astrophys. Jour., 13, 101, 1901.

rite window, as will be described later, and the whole instrument, except the window apertures, surrounded by a thick layer of wool, there was very little trouble from zero drift due to the ordinary temperature changes in the room.

The relation between sensitiveness and pressure seems to vary considerably in different instruments. In the one described by E. F. Nichols the sensitiveness is a maximum at about 0.05 mm. pressure, and the curve is rather flat at that point; while one used by G. W. Stewart,[1] which seems to be of similar construction, shows a quite sharp maximum at about 0.10 mm. The corresponding curve for this instrument (Fig. 1) may, therefore, be not without in-

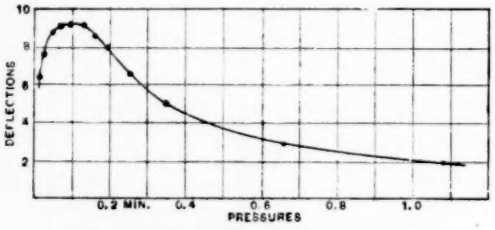

Fig. 1. Sensitiveness of Radiometer.

terest. The sensitiveness changes very little through the range of pressures from 0.04 mm. to 0.15 mm., which was quite convenient, as it proved impossible to prevent a slight leak. At pressures below 0.1 mm., the period is approximately forty-five seconds. At higher pressures, the swing is aperiodic, the full deflection being approximately reached in a time which increases slowly from forty-five seconds with the pressure.

The maximum sensitiveness is such that a paraffin candle burning normally at a distance of 200 cm. causes a deflection of about 30 cm. on a scale at a distance of 100 cm. This is about one half as great as that given by Nichols for the first instrument which he constructed, and about twice as great as for the one mentioned above.

Some of the readings with the vacuum-tube seemed to indicate that the larger deflections were not proportional to the energy, and a calibration of the scale showed this to be true. A 105-volt 16-c.p. incandescent lamp connected to a storage battery of 30 volts was used as a source, and placed at different distances, making sure

[1] PHYSICAL REVIEW, 13, 263, 1901.

that the radiometer was affected only by radiation which came from the lamp directly, and not by reflection. The results, plotted in Fig. 2, show that a deflection of 20 cm., which is the largest used, is about 12 per cent. too small. This large deviation is due doubtless to the fact that the vanes are so near the window, and especially that the one which is screened from radiation has a part of its surface opposed to the brass flange on the inside of the window, at a still less distance.

Vacuum-Tube. — Ångström used several tubes in his work, but they were all of nearly the same diameter. He suggested that it

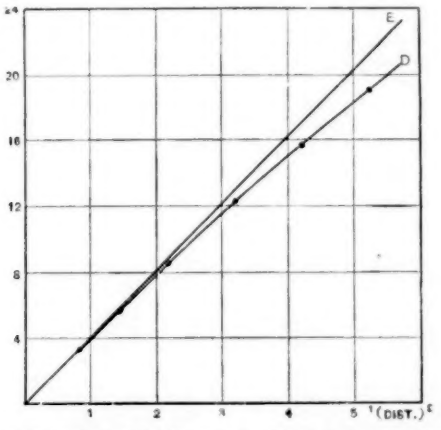

Fig. 2. Calibration of Radiometer. E, energy; D, deflection.

would be interesting to find the efficiency for tubes of widely differing diameters, and this was chosen as the first point of attack. The form of the tube, and the arrangement of tube, radiometer and screens, is shown in Fig. 3. The board to which the tube is secured is pivoted to a support at *A*, the intersection of the axes of the two branches, so that by swinging vertically about this pivot the axis of either branch may be brought into the horizontal line through the center of the radiometer vane. The smaller branch has an internal diameter of 9 mm., the larger of approximately 18 mm., and the length of the luminous column in each is about 10 cm. The aluminium electrodes in the side tubes are cylindrical, giving large surface, so that the heating is small. The ends of the two branches next the radiometer are closed by fluorite windows 1 mm.

thick. These, as well as all of the radiometer windows, are fastened with de Khotinsky cement.

A copper screen with a circular opening 4 mm. in diameter, 1 mm. larger than the vane, is placed about 3 cm. in front of the vane at *a*. Outside the radiometer case, about 5 cm. from the vane, is a larger double screen with a slightly larger opening. These serve to cut off from the vane all radiation except that which comes from the tube, and even with the smaller tube, a considerable share of the secondary radiation which may come from the heated tube walls. Between the end of the tube and the double screen space is left for the insertion of a cell *w*, which contains a layer of water 1 cm. thick between glass walls 1 mm. thick. Although this transmits more than the visible radiation, as did the alum plate used by Ångström, it still gives a definite division of the total radiation, which is sufficient for the immediate purpose in hand.

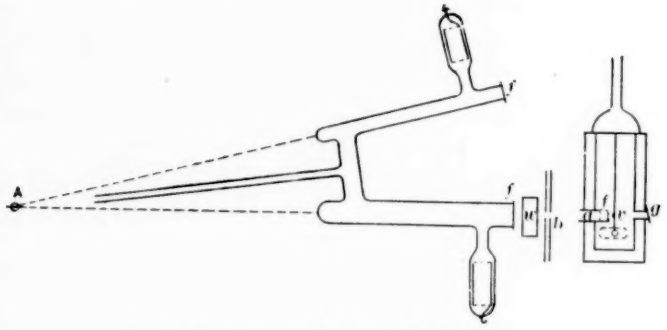

FIG. 3. Vacuum Tube and Radiometer.

Pressure Gauge, Pump and Connections. — For reading pressures a McLeod gauge was constructed in which the small tube is about 4 mm. in diameter, and the capacity of bulb 80 c.c. This enables pressures from 2 cm. to 0.01 mm. to be read with the necessary accuracy, and gives results which are consistent to within the error of reading, as well as can be judged by taking readings for the same pressure with different degrees of compression in the small tube.

Reduced pressure is obtained by means of a Geryk oil pump, with which pressures down to 0.01 mm. can be obtained easily and rapidly.

The gauge and pump, with the usual drying-tube containing phosphorus pentoxide, is connected to the vacuum-tube by a short

piece of rubber tubing, which is necessary to allow the required rotation of the latter, and by a longer piece to the radiometer, which can be closed from the rest of the apparatus by a glass stopcock. This rubber tubing, and the numerous joints, made it impossible to prevent leakage entirely. As air was the only gas used, and the leakage was small, it caused little trouble.

Experimental Details.

Alternating Current. — A large amount of preliminary work was done with an alternating current, chiefly on account of the ease with which it could be obtained. A city lighting circuit of 55 volts, 120 alternations per second, was connected through a rheostat to the primary of a large Charpentier induction coil, and the secondary connected to the terminals of the tube. It subsequently proved that the ratio of transformation of the coil was by no means constant, but varied with the resistance in the secondary circuit in a way which tended to keep the current constant.

Others who have used the radiometer have been troubled by electrification of the vanes, which usually occurs during the exhaustion, and persists for some time. When symptoms of this trouble appeared in the use of the present instrument, it was thought that the discharging action of radium might be utilized here by putting a few grains inside the radiometer. The result was a new and puzzling complication. There was still evidence of the usual electrification during exhaustion, which however quickly disappeared; but when a current was passed through the tube, there resulted what appeared to be an electrification effect, shown by a large shift of the zero, sometimes in one direction and sometimes in the other, which would disappear only after fifteen minutes or more. This made it impossible to determine the effect due to radiation alone. After some time spent in an attempt to discover the nature of the difficulty, and find a remedy for it, the radium was cleaned out as thoroughly as possible, and the effect soon became so small as to cause little trouble.

One of the difficulties experienced by Ångström was that due to radiation from the heated walls of the vacuum-tube, which sometimes contributed a large share to the effect on the bolometer. It was reasonable to expect that this effect would prove troublesome with

the smaller tube at least. It was tested several times with various currents and pressures, by screening the radiometer from the tube while the current passed for the usual time necessary to read a deflection, then removing the screen immediately after stopping the current and noting the deflection. This rarely amounted to more than one per cent. of the deflection which would have been produced by the current had the screen not been interposed. When the radiometer was exposed during the passage of the current the effect seemed to be somewhat greater, as nearly as could be judged by the time taken to return to zero with and without the water-cell interposed. The method of reading finally adopted to diminish these errors as far as possible is to pass current through the tube during 45 seconds, which usually gives a little less than the full deflection, then read the zero one minute after stopping the current. No shutter is used, as it would usually be at a temperature different from that of the tube, which is in the radiometer field during an exposure.

The first determinations of the efficiency ratio showed that it varied with the pressure in the way which would be expected, and that it was notably higher in the smaller tube. These results will not be quoted, as much more trustworthy ones were obtained later.

Besides the city circuit, there was available a University lighting circuit of 1,000 volts, 240 alternations per second. This was applied directly to the terminals of the tube, through a rheostat of cadmium iodide in amyl alcohol, and also transformed down to 50 volts, then transformed up again by the induction coil. The discharge through the tube differed greatly in appearance with these three kinds of current, and it seemed worth while to determine whether the efficiency would show corresponding variations.

a denotes city circuit, 55 volts, through induction coil.

b denotes 1,000 volts transformed down to 50, then through coil.

c denotes 1,000 volts on tube direct.

When the discharge through the small tube was viewed in a rotating mirror, or photographed on a falling plate, with (*a*) the band of light was almost continuous, a narrow dark space marking the interval between two successive discharges in opposite directions. With (*b*), the conditions being the same except that the frequency was twice as great, the dark space was wider in proportion to the

interval, showing that the discharge occupied a smaller fraction of the total time. With (*c*), the available potential difference at the electrodes of the tube was less, and the discharge would only pass at pressures less than 1 mm. The discharge now occurred at the crest of the e.m.f. wave, and occupied not more than one third the total time.

Several sets of determinations of the efficiency ratio failed to show any definite difference for these three cases. One set of such results is given below. Each result quoted is the mean of two determinations, for which the order of reading radiometer deflection, with and without the water-cell interposed, is reversed.

TABLE I.

Efficiency with different kinds of alternating current. Small tube. Pressure of air between 0.4 *mm. and* 0.5 *mm.*

a	*b*	*c*
.508	.480	.497
.502	.501	.521
.507	.492	.509
.511	.524	.519
.509	.529	.527
.513	.498	.512
.509	.485	.509
Mean .508	.501	.513

The slight differences among the mean results are hardly greater than the probable error.

An attempt to measure the current through the tube proved so unsatisfactory that it is worthy of description. A Weston alternating current voltmeter reading to 600 volts, resistance 21,600 ohms, was the only instrument at hand which would give readable deflections when used as an ammeter in series with the tube. In this type of instrument the lower part of the scale is much contracted, and the error of reading may be as great as five per cent. The deflections showed no evidences of electrostatic disturbances. For the three kinds of current, readings were taken of the current as given by the ammeter, and of the radiometer deflection for the total radiation from the tube. A preliminary trial showed that these are approximately proportional for one kind of current; and as it was

not always convenient to obtain the same current, this proportionality was used to reduce radiometer deflections to the same ammeter reading.

The effect of the current in producing radiation in the tube is thus not at all proportional to the electrodynamometer effect as shown by the voltmeter. The appearance of the discharge when viewed in a rotating mirror indicates that the wave-form may be quite different in the three cases. The voltmeter may not read correctly for currents in which the wave differs greatly from the simple sine form; and it is to be noted also that the radiometer effect is proportional to the mean first power of the current, and the voltmeter reading to the square root of the mean square.

TABLE II.

Radiometer and ammeter effects of current. Pressure 0.4 *mm.*

	Ammeter Current.	Radiometer Deflection.	Deflection Reduced to Current 0.008.
a	0.0062 amp.	11.3 cm.	14.6 cm.
b	0.0081	12.5	12.4
c	0.0079	8.8	8.9

Direct Current. — The subsequent work was done with a direct current chiefly, in part because of these difficulties in measuring the alternating current, and also because any considerations based on the electron theory would require a knowledge of the potential gradient in the tube at the time of taking the radiometer readings. The potential gradient would have no very definite meaning in the case of the alternating current, nor could it be readily measured.

The available source of direct current is a battery of twenty-four small Crocker-Wheeler dynamos, each giving 500 volts, connected in series. Eight of these were ordinarily used, giving about 4,000 volts, which is sufficient to send the discharge through the tube at pressures up to about 5 mm. The current is read by a Weston standard milli-voltmeter, resistance 9.06 ohms, used as an ammeter, on which a current of 0.001 ampere gives a deflection of 4.5 scale-divisions approximately. The current usually employed, 0.007 amperes, can be easily read to less than one per cent. The end of the circuit which contains the ammeter is grounded to prevent

electrostatic disturbances, which without this precaution would often cause the needle to stick. In the cadmium iodide rheostat which is included in the circuit the upper electrode is suspended in the tube by a cord, so that its height can be readily varied as necessary in order to keep the current at the desired value. There are small irregular changes due probably to variation in the speed of the dynamos, and also an increase in current for a few seconds on first starting current through the tube, which may be due to something in the nature of a resistance which decreases with rise of temperature, since it is proportionally larger with larger currents.

The two deflections of the radiometer, with and without the water-cell, from which an efficiency ratio is calculated, must correspond to the same total radiation, so this adjustment to keep the current constant is quite important. The fluctuations can usually be kept within one per cent. The sensitiveness of the radiometer does not change appreciably during the short time covered by the two readings, nor does the pressure in the tube, except at very low pressures, as will be mentioned later.

The usual telephone method for testing the steadiness of the current through the tube was not very satisfactory, as there is always a faint hum in the telephone caused by the minute fluctuations in e.m.f. at the commutators on the dynamos. There was no fluctuation other than this, so far as could be judged by comparison with the effect produced when the current flowed through the rheostat only, without the tube. The resistance outside the tube was never greater than 8×10^5 ohms; and Hittorf[1] found no evidence of unsteadiness in the discharge, under similar conditions, until the outside resistance reached 3.5×10^6 ohms.

Efficiency — Variation with Pressure, and with Tube Diameter.— The series of observations for finding the efficiency as a function of the pressure, in the two tubes separately, is given in Table III. The observed radiometer deflections, after correction from the calibration curve, Fig. 2, are tabulated in the order in which they were taken. The same current-strength was used throughout. Series I. and II. were taken on different days. In series III. are collected a number of single determinations which were made, under comparable con-

[1] Wied. Ann., 7, 563, 1879.

TABLE III.

Variation of Efficiency with Pressure. a, Corrected Radiometer Deflection for Total Radiation. b, Corrected Radiometer Deflection for Water-Cell Radiation. $e = b/a$, Efficiency Ratio. p, Pressure in Millimeters. Current, 0.007 Amperes.

a	b	a	e	Mean, e	p
Small Tube.—Series I.					
	4.77	20.40	.234		
19.92	4.80		.241	.237	4.3
	5.48	19.07	.288		
18.87	5.37		.285	.286	3.3
	5.75	18.16	.317		
17.60	5.65		.321		
	5.56	17.50	.318	.319	2.6
	5.41	17.41	.311		
17.00	5.44		.320	.315	2.7
	6.08	16.92	.360		
16.56	5.94		.359		
	5.94	16.56	.359	.359	2.14
16.86	5.90		.350		
	5.71	16.43	.348		
16.50	5.88		.351	.350	1.9
	6.52	17.31	.376		
16.70	6.38		.382	.379	1.52
16.58	6.42		.388		
	6.21	16.36	.380	.384	1.25
17.11	6.88		.402		
	6.74	17.15	.393	.397	1.0
	9.01	19.90	.453		
20.55	9.05		.441	.447	0.66
	8.94	18.98	.471		
18.67	9.07		.486	.478	0.46
Series II.					
22.75	9.25		.408		
	8.87	22.26	.399	.403	1.12
23.00	9.75		.425		
	9.58	23.30	.412	.418	0.85
20.37	10.00		.492		
	9.88	20.61	.480	.486	0.40
Series III.					
	mean	of 7		.262	3.85
11.83	3.87		.327		
	3.82	11.92	.320	.323	2.95
6.46	2.00		.310		
6.38	1.99		.312		
	1.97	6.40	.308	.310	2.95

TABLE III.—*Continued.*

a	*b*	*a*	*e*	Mean, *e*	*p*
Large Tube.—Series I.					
	0.66	9.56	.069		
8.97	0.54		.060	.064	4.3
8.65	0.51		.059		
	0.53	8.00	.056	.062	4.4
	0.56	5.90	.095		
5.66	0.59		.104		3.3
5.62	0.57		.101	.100	4.4
4.50	0.57		.127		
	0.56	4.27	.132		
4.32	0.55		.127	.129	2.7
3.06	0.41		.134		
	0.43	3.19	.135	.135	2.36
3.04	0.46		.152		
	0.46	2.96	.156	.154	1.84
	0.52	2.48	.210		
2.38	0.49		.206	.208	1.56
	0.56	2.28	.245		
2.16	0.56		.259	.252	1.23
1.94	0.60		.309		
	0.60	1.98	.303	.306	1.02
2.02	0.68		.336		
	0.66	2.01	.328	.332	0.66
	1.31	2.79	.470		
2.87	1.28		.446	.458	0.46
Series II.					
	0.43	1.69	.255		
1.73	0.41		.237	.246	1.23
3.08	0.81		.263	.263	1.10
2.68	0.82		.306		
	0.78	2.60	.300	.303	0.85
2.76	0.92		.334		
	0.87	2.75	.316	.325	0.68
3.16	1.21		.382		
	1.19	3.22	.370	.376	0.52
Series III.					
4.05	0.40		.099		
	0.40	3.58	.113		
3.69	0.38		.103	.105	3.1
6.81	0.86		.126		
	0.87	6.70	.130		
6.62	0.89		.134	.130	2.6

ditions, during later work. The current was always sent through the tube so that the branch used was next the anode. Usually observations were made on the two branches alternately, to insure similarity of conditions.

Immediately after Series I. and II., a similar set of readings was taken with the alternating current from the city circuit. It was assumed that the radiometer sensitiveness had not changed appreciably, and the radiometer effect was taken as an appropriate measure of the current. At each pressure then the current was adjusted, by means of the rheostat in series with the primary of the induction-coil, until the radiometer deflection for the total radiation was nearly the same as had been observed with the direct current. The

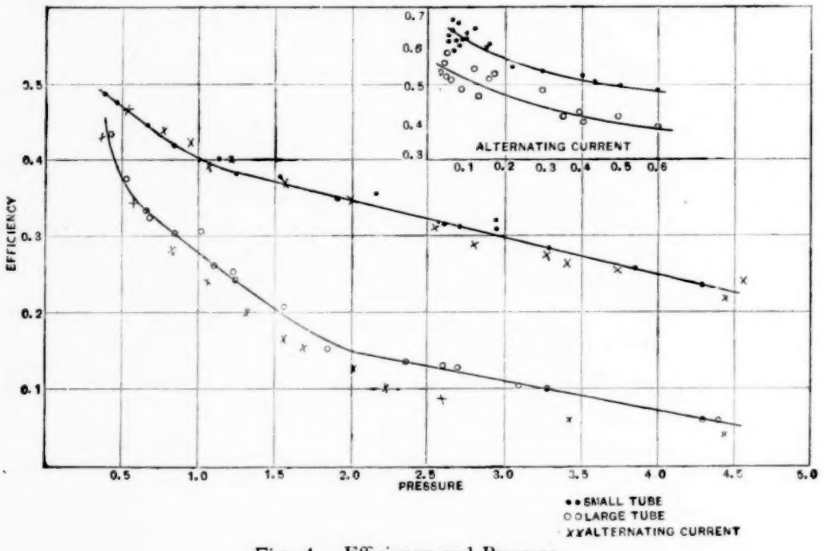

Fig. 4. Efficiency and Pressure.

measurements were otherwise carried through in precisely the same way as with the direct current.

Fig. 4 shows the direct current results graphically; and the alternating current results are added for comparison.

The direct current gives a continuous luminous column filling both branches of the tube at the higher pressures. When the pressure is reduced, the column in the small tube shows faint hazy striation at about 2 mm., which fades out at 1.5 mm., when this end is anode. At about 1 mm. the striæ reappear, at once become

sharply defined, and remain to the lowest pressures used. When the anode is next to the large branch, striation begins there at about 1.6 mm., and becomes gradually more distinct as the pressure is reduced. With this direction of current the striation in the small tube does not disappear, but on the contrary is quite sharp and distinct through the range from 1.5 mm. to 1 mm.

With this tube it has proved impossible to keep the current steady enough to secure trustworthy results at pressures below 0.3 mm. This seems to be due to the fact that the enlarged portion of the side tube which contains the negative electrode becomes entirely filled with the blue cathode glow, which at irregular intervals extends itself through the narrow connection into the main tube, and greatly reduces the current. This lasts for a few seconds only, when the normal conditions return.

This difficulty did not occur at all with the alternating current. Below 0.15 mm., however, the pressure decreases during the passage of the current, probably on account of the absorption of the gas by the walls, which was investigated by Willows;[1] and as the radiometer deflection here falls off rapidly with the pressure, this becomes a serious difficulty. The small plot in Fig. 4 gives the results of a large number of readings with the alternating current in this region, each point representing several determinations. It has little value except to show the general nature of the curve; and even this is doubtful below 0.1 mm.

A possible explanation of the lower values found with the alternating current, as shown by Fig. 3, is that the efficiency is different for the two directions of current in the tube used, being greater when it is next the anode, as used with the direct current. A series of readings with the large tube to test this point gave the following results on page 248.

Current reversed means the reverse of the direction ordinarily used. The efficiency proves to be somewhat less at the cathode end, but with the alternating current it is lower still, so this explanation is not sufficient.

Another possible reason for the difference is suggested by the fact, previously mentioned, that with the alternating current there

[1] Phil. Mag. (6), 1, 503, 1901.

seems to be no current through the tube during a portion of the time. The current while flowing would thus have a larger mean value than that of the direct current which produces the same radiometer effect; and the results next presented show that the efficiency is in general smaller for larger currents, this being more noticeable in the large tube for the range of current here used.

TABLE IV.

Effect of Direction of Current.

Current Reversed.	Current Direct.	Current Reversed.	Current Alternating.
.090	.099	.090	.087
.082	.113	.100	.083
.105	.103	.094	.086
Mean .092	.105	.095	.085

Pressure 3.1 mm. Current 0.007 ampere.

Variation with Current.— Ångström [1] found that with nitrogen the efficiency was independent of the current strength through a rather wide range. Several trials through a small range seemed to show the same result for this tube. As some theoretical considerations however seemed to point to the existence of such a variation, a more careful trial was made through a wider range. This showed its existence, and also the reason why it had been overlooked before. The results for the two tubes are given in Table V., and plotted in Fig. 9.

TABLE V.

c, Current in milliamperes. e, Efficiency.

		Pressure 3.85 mm. Small Tube.		Pressure 2.6 mm. Large Tube.	
c	e_s	c	e_s	c	e_l
7.0	.266	3.5	.256	7.0	.130
1.6	.290	1.6	.293	10.5	.130
10.5	.290	7.0	.264	3.5	.156
7.0	(.278)	4.5	.246		
	(.260)	7.0	.258		

The earlier trials had been made with the small tube in the neighborhood of 6 milliamperes where the variation happens to be small.

[1] Wied. Ann., 48, 508, 1893.

The two bracketed values were obtained for the same current under different circumstances. For the first, $e = .278$, the readings were taken immediately after the tube had been used with a stronger current, which had heated it considerably. After cooling for five minutes the readings were taken which gave the smaller result. This suggests that the rise of temperature which accompanies increased current may increase the efficiency; and with the larger currents this may over-balance the decrease due to the effect of current change alone.

Variation with Temperature. — It seemed desirable to test this temperature effect by heating the tube from the outside and using a constant current. There was no obvious way to apply the heat without a radical change in the mounting of the tube; but a satisfactory result was finally obtained by using the current as a source of heat. A wooden box was fitted over the small tube, and the joints packed with wool. A current of about 12 milliamperes would heat the air in the box enough so that after stopping the current and allowing the tube to cool a few minutes to secure uniform temperature throughout, the whole would be about 20° above room temperature. The readings were then taken with the usual current of 7 milliamperes, which was sufficient to maintain the increased temperature for the necessary time. A similar determination was then made with the air in the box at room temperature, so that the conditions are the same in the two cases, except for the difference in temperature of the medium surrounding the tube. The results are:

TABLE VI.

Pressure 2.8 mm.

Temperature.	
40°	$e = .329$
21°	.323

A similar determination made several weeks later gave almost identical results. The method could not be used at lower pressures, as the heating effect of the current is not sufficient.

Total Radiation. — A special species of readings, without the water-cell, was taken to show the total radiation from the tube as a function of the pressure and of the current. The readings were taken in the order shown in Table VII. as a check against change

in sensitiveness of the radiometer or other conditions. It must be assumed that the radiometer deflection is always proportional to the total radiation per unit length of the luminous column. This is more likely to be true with the small tube than with the large, since in the latter there are more appreciable variations in the size and general appearance of the luminous column at the points of entrance and exit, as the pressure or the current is changed. The results for the large tube are less trustworthy for this reason; and in fact it was impossible to obtain any consistent readings for the variation with current.

TABLE VII.

Total radiation with varying pressure, and with varying current.

Deflection and Pressure.

Small Tube.				Large Tube.			
p	*d*	*p*	*d*	*p*	*d*	*p*	*d*
1.6	8.00	0.62	8.47	1.12	5.92	0.33	4.75
2.56	8.39	0.39	8.03	2.25	7.97	0.22	5.77
3.85	9.09	1.9	8.34	2.72	8.70	0.13	5.28
4.5	9.55	0.56	8.62	4.25	11.60	3.4	10.13
3.36	8.98	0.32	8.29	2.90	8.70	1.2	6.04
2.50	8.47	0.18	7.23	2.2	7.97	0.44	5.29
1.9	8.28	0.10	4.70	1.6	6.80	0.9	5.34
1.4	8.18	0.06	3.29	1.28	6.10	0.56	5.00
1.04	8.53	0.045	2.45	1.00	5.40	0.24	5.81
0.8	8.48			0.8	4.80	0.07	3.50
				0.5	4.13	0.04	2.60

Deflection and Current. Small Tube.

c	*d*	*c*	*d*
1.6	6.16	3.5	10.40
7.0	16.85	1.6	5.80
10.5	21.60	4.5	12.76
7.0	16.74	7.0	16.80

The results for change of pressure are plotted in Fig. 5.

Total Radiation — Variation with Size of Tube. — If it may be assumed that at a given pressure the efficiency as measured by the water-cell is the same multiple of the true luminous efficiency in the two tubes, a method suggests itself for comparing the total radiation per unit length of the two tubes. This cannot be done with the radiometer directly, for different fractions of the total radiation

reach the radiometer vane in the two cases. It is only necessary to measure the ratio of luminous intensities by a photometer method, and reduce this to ratio of total intensities by using efficiency ratios.

A paraffin diffusion photometer was used for this purpose, made of two blocks of clear paraffin 2 mm. thick and 14 mm. long, separated by tin-foil. This was arranged to move in the space between the tubes so that the radiation fell upon its two faces at nearly equal angles at the final setting. The tubes were covered with black paper, leaving apertures through which one centimeter, as nearly as

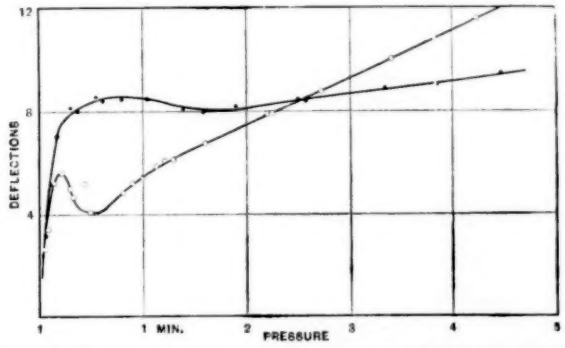

Fig. 5. Total Radiation and Pressure. Dots, Small Tube; Circles, Large Tube.

possible, of the axis of each tube was visible from a point at the center of the photometer block.

The pressures were so chosen that the luminous column in each tube was entirely continuous, and the difference in color apparently as little as possible. The alternating current from the city circuit was used. The efficiencies for the large tube, determined at the time, are somewhat higher than those shown in Fig. 4, but correspond to those of Table IV. Each result is the mean of seven settings of the photometer block, which agreed to within 0.5 mm. The distances l and s, Fig. 6, are measured from the tin-foil partition to the axes of the tubes.

The effect of the total radiation from one tube which reaches the photometer block is nearly the same as though it were all concentrated in the axis. For consider two layers at equal distances $+ x$ and $- x$ from the axis (Fig. 6). Their distances from the photometer block are $l + x$ and $l - x$; and their lengths are also propor-

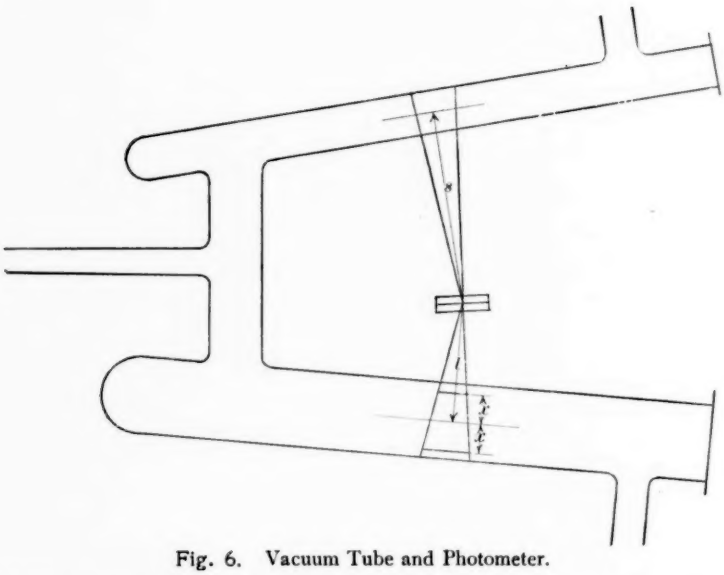

Fig. 6. Vacuum Tube and Photometer.

tional to those distances. The assumption then amounts to saying that

$$\frac{l+x}{(l+x)^2}+\frac{l-x}{(l-x)^2}=2l/l^2$$

For the extreme case when $x=r$ for the large tube the error is less than 5 per cent. The ratio of the total radiation per unit length for the two tubes is then computed as

$$\frac{R_l}{R_s}=\frac{l^2e_s}{s^2e_l}.$$

TABLE VIII.

ϕ (mm.)	l	s	e_l	e_s	R_l/R_s
2.3	3.25	4.85	.135	.335	1.08
2.75	3.10	5.00	.105	.300	1.10

The probable error of the results is doubtless as great as ten per cent. and to this approximation the ratio is unity.

Potential Gradients. — As the tube previously described had not been planned for the measurement of potential gradients, another was obtained of the same general form and as nearly as possible of

the same dimensions. This had a fine platinum wire stretched across the middle of each branch of the tube, perpendicular to its axis, and a similar wire on each side of this, at distances of approximately 3 cm. The measurements were made by the method of condenser and ballistic galvanometer.

The condenser, a standard by Nalder Bros., has a range from 0.05 to 1.0 microfarad, which made it possible to keep the galvanometer throws within the range from 4 to 12 cm., although the differences of potential measured varied in the ratio of 1 to 16.

The D'Arsonval galvanometer used was much too sensitive, so its coil was shunted by means of a piece of German silver wire. This served the additional purpose of short-circuiting the coil and quickly damping its vibration after the condenser had been discharged through it. The complete period was five seconds, and the damping so good that three readings could easily be obtained within forty-five seconds. The resistances of coil and shunt were afterward measured and found to be in the approximate ratio of 1 to 20, while the throws for equal quantities were about as 1 to 40. The back e.m.f. due to the beginning of the coil's motion thus greatly modifies the distribution of the current.

The galvanometer scale was calibrated directly by charging the condenser from a storage battery to a known difference of potential, measured by a voltmeter, and then discharging through the galvanometer. The data given below show that the deflection was quite accurately proportional to the potential difference for the range used, and also give an idea of the accuracy of the readings.

Calibration of Ballistic Galvanometer and Condenser.			Condenser Charged to 76.7 Volts.
Capacity.	1.0	0.5	0.2 + 0.2 + 0.05 + 0.05
Throws.	9.72	4.83	4.85
	9.71	4.85	4.84
	9.69		

For capacity of 1 microfarad, 1 cm. throw = 7.9 volts.

The system of connections used, with the ordinary three-way discharge key, made it necessary to keep one galvanometer terminal continuously connected to the condenser, and so to the tube. When the cathode was at the grounded end of the line, this gal-

vanometer terminal was raised to a potential high enough to cause a slight electrostatic action, which resulted in a slow zero drift. The uncertainty thus introduced into the readings was not often greater than one per cent. It was seldom noticed when the anode was next the grounded end of the line, as the chief drop of potential within the tube occurs at the cathode.

Connecting wires from the six platinum wires in the tube led to mercury cups in a block of paraffin made it easy to connect any desired pair of wires to the condenser terminals.

The attempt was now made to determine the potential gradient under all of the various sets of conditions for which the efficiency had previously been found. Aside from accidental variations in the composition of the air used, the only evident difference in conditions is shown by the fact that the appearance of the discharge is slightly modified by the introduction of the wires. In the unstriated discharge, each wire is accompanied by a faint stria. When the pressure is reduced until striation begins, each stria behaves as though covered by an elastic surface film, which the wires penetrate with difficulty, and hence the striæ are distorted in form, and displaced in position, where they come in contact with the wires. This was noticed by Wood.[1] Reducing the pressure still further until the striæ become hazy in outline, this effect disappears, and the wires have no perceptible effect upon their distribution or form. The change in potential gradient which accompanies this changed appearance is of course unknown; but the mean value for the whole tube, which is the desired result, is probably little affected.

In the tabulated summary of results, Table IX, V_a is the potentila gradient calculated from the observed potential difference between wires 1 and 2, V_b between wires 2 and 3, V_{a+b} between wires 1 and 3; wire 1 being the one nearest the electrode which is next the tube used. The differences between results for the different intervals are but little greater than the probable error of observation. Such consistent results were hardly expected in the striated discharge, where the results of Graham[2] and H. A. Wilson[3] show usually a change from point to point, in passing from one stria to

[1] Wied. Ann., 59, 250, 1896.
[2] Wied. Ann., 64, 49, 1898.
[3] Phil. Mag. (5), 49, 505, 1900.

the next, as well as a progressive increase in going along the tube from anode toward cathode. There is some evidence of the latter variation in the small tube. The mean result for each case probably corresponds nearly enough to the mean potential gradient in the discharge which affected the radiometer.

TABLE IX.

Potential Gradients.

Small Tube.

V_b	V_{a+b}	V_a	Mean.	p	V/p
6.40	63.7	64.4	64.0	3.12	20.5
72.5	72.1	71.4	72.0	3.68	19.6
79.6	80.3	78.8	79.6	4.26	18.7
67.8	66.4	65.6	66.6	3.22	20.7
55.8	55.1	54.1	55.0	2.46	22.4
45.3	45.5	45.3	45.4	1.88	24.1
38.3	37.7	39.1	38.4	1.42	27.0
32.9	32.7	32.0	32.5	1.09	29.8
29.4	28.5	28.1	28.7	0.83	34.6
21.9	21.6	21.3	21.6	0.48	44.5
51.7	51.5	51.4	51.5	2.28	22.6
25.6	25.9	25.6	25.7	0.81	31.7
24.2	23.9	24.3	24.1	0.60	39.6
21.2	20.8	20.2	20.7	0.46	45.0
15.9	16.0	16.3	16.1	0.28	57.5
13.3	12.8	12.4	12.8	0.19	67.5

Current, m. a.		Mean V.	p	V/p
	1.6	91.5	3.68	24.7
	3.5	80.6		21.8
	4.5	76.5		20.7
	7.0	71.0		19.3
	10.5	68.0		18.4
Striated Discharge	11.	38.3	1.54	
	5.5	41.0	"	
	11.	24.3	0.71	
	5.5	26.3	"	

Temperature.	V	p
20°	58.0	2.74
41	57.5	
20	59.0	
40	57.6	

TABLE IX.—*Continued.*

Large Tube.					
V_b	V_{a+b}	V_a	Mean.	p	V/p
55.3	55.8	55.5	55.5	3.12	17.8
63.0	63.5	62.3	62.9	3.68	17.1
68.6	68.1	68.3	68.3	4.26	16.0
56.0	56.7	57.7	56.8	3.22	17.6
47.6	48.3	48.2	48.0	2.46	19.5
39.3	39.8	39.9	39.7	1.88	21.1
32.7	32.1	32.4	32.4	1.42	22.8
27.6	27.6	27.6	27.6	1.09	25.4
23.4	22.9	22.4	22.9	0.83	27.6
15.8	15.8	15.5	15.7	0.48	32.2
39.5	39.2	39.8	39.5	1.79	22.0
27.3	26.9	27.9	27.1	1.06	25.5
19.3	18.7	19.0	19.0	0.61	31.1
14.4	13.8	13.7	14.0	0.38	36.4
11.1	11.1	11.2	11.1	0.23	47.3
8.5	8.4	8.4	8.4	0.16	53.0

Current, m. a.	V	p	V/p
3.8	58.2	2.64	22.1
4.5	56.0		21.2
7.0	52.4		19.3
10.5	47.7		18.1

These results are in general agreement with similar ones obtained by Herz[1] and Schmidt[2] for nitrogen, with the single exception that the latter finds, in the case of the striated discharge, an increased potential gradient with increasing current, while the opposite is true in the present case.

DISTRIBUTION OF ENERGY IN THE SPECTRUM.

Through the kindness of Mr. W. W. Coblentz I was enabled to use his spectrometer, with rock-salt prism and radiometer,[3] in the attempt to learn something about the distribution in the spectrum of the energy which affects the radiometer. The energy is so small that it was necessary to use wide slits in front of the tube and the radiometer window in order to get readable deflections, and the consequent overlapping, with a spectrum of this character,

[1] Wied. Ann., 54, 244, 1895.
[2] Drude's Ann., 1, 625, 1900.
[3] PHYSICAL REVIEW, 16, 44 and 72, 1903.

makes it impossible to do more than find the approximate location of the principal maxima, and get a general idea of the distribution. The small tube was used, and the pressure kept at approximately 0.9 mm. The heating effect is small at this pressure, so that a stronger current could be safely used, and a small pressure change causes little change in the amount of the total radiation. The alternating city circuit was used for convenience, so its strength could not be readily measured. It was probably as much as 15 milliamperes. As the collimator was moved to throw different regions of the spectrum on the radiometer slit, the vacuum tube had to be adjusted by hand, which may have caused some uncertainty in the readings; and the current was by no means constant.

The curve, Fig. 7, shows the observed deflections in millimeters on a scale at 150 cm., wave-lengths being obtained from the cali-

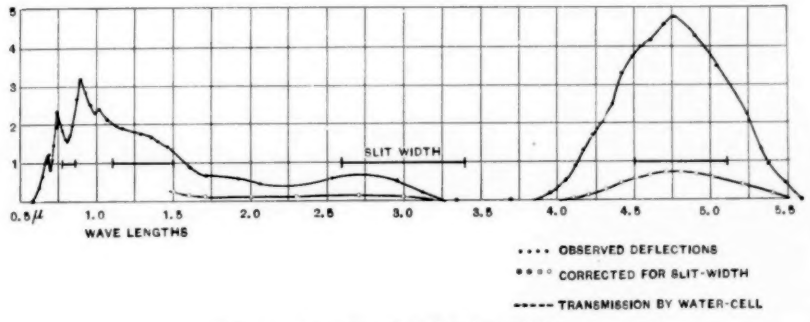

Fig. 7. Spectrum Distribution of Energy.

bration curve prepared by Mr. Coblentz. Fig. 8 shows the region out to 1.5 μ, plotted on a larger scale of dispersion. The zero was nearly always unsteady, which made the smaller readings uncertain. Each point represents the mean of three or more readings, taken at favorable times when the conditions seemed best. The whole region as far as 7.5 μ was covered consecutively in one day; and some days later the region from 0.6 μ to 1.5 μ was explored again, with results almost identical as regards the location and relative magnitude of the maxima.

The curve shows well-defined maxima at wave-lengths 0.66, 0.74, 0.89 and 4.75; while less prominent ones are indicated at 1.02, 1.40, 2.75 and 4.4.

Each of the slits was 1 mm. wide; and this width, expressed in wave-lengths, is shown by a horizontal line, for various regions of the spectrum. The observed curve is corrected for slit-width by means of the formula given by Paschen,[1] using the first two terms of the series. The ordinates to this corrected curve are chosen merely to give a convenient scale.

If it be assumed that this corrected curve properly represents the distribution of energy, it is of course possible to compute the true luminous efficiency, as the ratio of that portion of the area to the left of the ordinate 0.76 μ to the whole area. The reliability of this

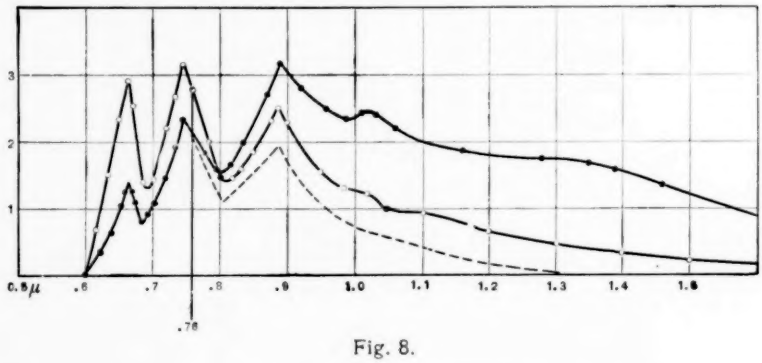

Fig. 8.

result may be roughly checked by computing in a similar way the water-cell efficiency, and comparing this result with that obtained by the direct method. For this purpose Prof. E. L. Nichols has kindly allowed me to use some unpublished data on the transmission of the water cell, which show that it is approximately 0.85 through the visible spectrum, and then falls to nearly 0 at 1.5 μ. The dotted curve is obtained by applying these transmission ratios to the ordinates of the corrected energy curve from 0.76 μ on. Then the area included under this curve, plus eighty-five per cent. of the visible energy previously obtained, is taken as the energy which would get through the water cell.

The results are 0.18 for the luminous and 0.42 for the water-cell efficiency. These are too low, however, because the losses due to absorption and diffusion in passing through the rock-salt prism, and to reflection at the silvered mirrors, are all greater for the

[1] Wied. Ann., 60, 712, 1897.

shorter wave-lengths. The only available data for determining the rock-salt loss are some observations by Mr. Coblentz on a thin plate, with surfaces in about the same condition as those of the prism, as regards lack of polish. These show a diminished transmission of about twelve per cent. from 4.5 μ to 1 μ and of nearly twenty per cent. to 0.7 μ. A detailed correction of the curves on this basis is hardly warranted. A reasonable estimate, including the effect of the silvered mirrors, is that the water-cell efficiency as given above should be increased about ten per cent., and the luminous efficiency about fifteen per cent., which would make the numbers forty-six and twenty-one respectively.

This result for the water cell is apparently too large, when compared with the value 0.41 given by the curve in Fig. 4; but on account of the stronger current here used the efficiency would be increased to a value perhaps even higher than this, according to the lower curve of Fig. 10. The two results are thus in agreement as nearly as can be determined. It then follows that 0.20 is a good approximate value for the luminous efficiency under the working conditions, which is at least not too large.

It is to be noted that in the curve of observations, Fig. 7, the maximum at 4.75 μ has very nearly the form which would be given by a single line in the spectrum at that wave-length. It is approximately a triangle, whose base is twice the slit-width. Nearly the whole energy in this portion of the spectrum thus seems to be concentrated in a single line, or at least a narrow band or group of lines.

In testing the practicability of a suggestion that a carbon bisulphide prism might be used to advantage with this radiation, it had been found that a layer of carbon bisulphide not more than 3 mm. thick, between quartz plates, produced an absorption comparable with that of the water cell. This is now explained by the fact that carbon bisulphide has a strong absorption band at about 4.7 μ, as was found by Mr. Coblentz in the course of an investigation soon to be published.

This radiation comes from air, which contains traces of mercury, from the pressure gauge, and of hydrocarbons, from the oil in the pump and the grease in the joints.

THEORETICAL.

According to the electron theory in its present form, the ordinary gas molecule is thought of as an aggregate of particles carrying electric charges, which must be positive and negative in equal amount, since the molecule as a whole is electrically neutral. It is possible to separate from the molecule a minute particle which carries a negative charge, and thus leave with the remainder of the molecule an equal positive charge. This negatively charged particle, or electron, carries a charge equal to that of the hydrogen ion in electrolysis, as was shown by Townsend.[1] From the measured value of the ratio of charge to mass, and the magnitude of the charge, which has recently been determined by J. J. Thomson[2] and H. A. Wilson,[3] the computed mass of the particle is about 1/700 as great as that of the hydrogen atom. And this is independent of the kind of matter from which it comes.

Whether the remainder of the molecule is an aggregate of similar small particles is still an open question, so far as the writer is aware; and it is perhaps immaterial for our present purpose.

We are then to think of the positive column of our vacuum tube, when the current has been established through it, as occupied by gas molecules, a certain small number of which have been dissociated in this way. These dissociated particles will have, beside their ordinary gas motion, a component motion along the length of the tube, produced by the electric forces. The positive and negative charges being equal, the forces will be the same; but the negative will acquire much greater speed in the same distance, on account of the smaller mass with which they are associated. Each will, however, acquire the same energy in moving through the same difference of potential.

This motion will be frequently interrupted by collisions with neutral molecules which are moving at random. The mean free path will not differ much from that calculated by the kinetic gas theory, for the positively charged particles. It may be a little greater, since their speed will be greater than that of the neutral molecules, as determined by the temperature, and so they approach

[1] Phil. Trans., 193, 153, 1900.
[2] Phil. Mag. (6), 5, 346, 1903.
[3] Phil. Mag. (6), 5, 429, 1903.

somewhat toward the condition of particles moving through a field of similar particles at rest. The negative electrons will have a longer free path, because of their smaller size ; but it probably cannot be computed directly, since the size here means the diameter of the sphere of action, and this differs from that considered in the kinetic theory, in that it depends upon the electric forces chiefly.

The mean energy at collision, in any case, will be that acquired by the charged particle in moving through its path under the action of the electric forces. When such a collision occurs, a part of this energy will be transferred to the molecule struck, where it will in general produce two effects. The first is an increase in the kinetic energy of the molecule as a whole, which, considering all such effects, means a rise in the temperature of the gas. The second is an effect within the molecule itself, producing relative motion among its parts. As these parts carry positive and negative charges, and are probably held together in a coherent system by the electric forces acting between them, such relative motion will be vibratory in character, and hence a probable source of electromagnetic radiation. The intensity of such radiation would vary with the energy of collision, which is greater for the negatively than for the positively charged particles. J. Stark [1] has suggested that the negative electrons, on account of their smaller size, are able to act selectively on the component parts of the molecule, and so cause much greater vibratory motion compared to the motion of the whole molecule, than the positive particles ; so that the latter may be left out of account in considering intensity of radiation. Whether this is true or not will make little difference in our application of the theory.

It is of course possible that a collision may occur with sufficient energy to cause dissociation of the molecule struck, thus producing a new pair of charged particles. Townsend has succeeded in measuring the number of such dissociations produced by a negative electron in passing through one centimeter of gas, with various pressures and potential gradients. The results applied to the conditions which obtain in the positive column of a vacuum tube [2] show that the proportion of collisions which produce dissociation is exceedingly small, so that this feature of the case need not be considered.

[1] Die Electrizität in Gasen, Stark, p. 444.

[2] Phil. Mag. (6), 1, 226, 1901.

According to this view, radiation from a gas due to an electric current through it, differs from that due to high temperature chiefly in the fact that the source of the radiation, namely the vibratory motion within the molecule, is caused in the latter case entirely by collisions between the molecules, while in the former case it is caused by collisions between molecules and charged particles, which are for the most part much smaller than molecules, and are moving at relatively high speeds. In the first case the energy increases with the temperature, in the second with the potential gradient; the density of the gas being supposed constant.

Since with high temperature radiation the efficiency, or proportion of high-frequency radiation, increases with the temperature, or with the energy of the collisions, it seems likely that the efficiency of the electrical radiation should also increase with the energy of collision, which is measured by the product of potential gradient and mean free path. J. Stark[1] has proposed this view, and applied it to Ångström's results. It will now be applied to the results previously described.

Variation with Pressure. — The energy of collision will vary with Va, where V is potential gradient and a is mean free path. If the temperature of the gas be assumed to remain constant and the pressure to vary, Va will vary with V/p.

In Fig. 9 the values of V, and also of V/p, are plotted for the two tubes. Comparison of the latter with the efficiency curves of Fig. 4 shows that the variation is in the right general direction for each tube separately. If, however, the efficiency is a function of Va only, a given change in the value of Va, no matter how produced, should cause the same change in the corresponding efficiency ratio. Starting with Va or V/p, in the large tube at 3 mm. pressure, its value may be increased by going to the small tube at the same pressure, or by reducing the pressure in the large tube; and equal changes produced in this way are not accompanied by equal changes in the efficiency ratio. This point will be referred to again, after some other cases have been considered.

Variation with Temperature.—Since the tube is in connection with the much larger volume of gauge and pump, the pressure re-

[1] Electrizität in Gasen, p. 450.

mains nearly constant when the temperature rises. The mean free path will then be determined by the density, and its change may be calculated from the temperatures. These are unfortunately not known under the conditions of the experiment; if Wood's results [1]

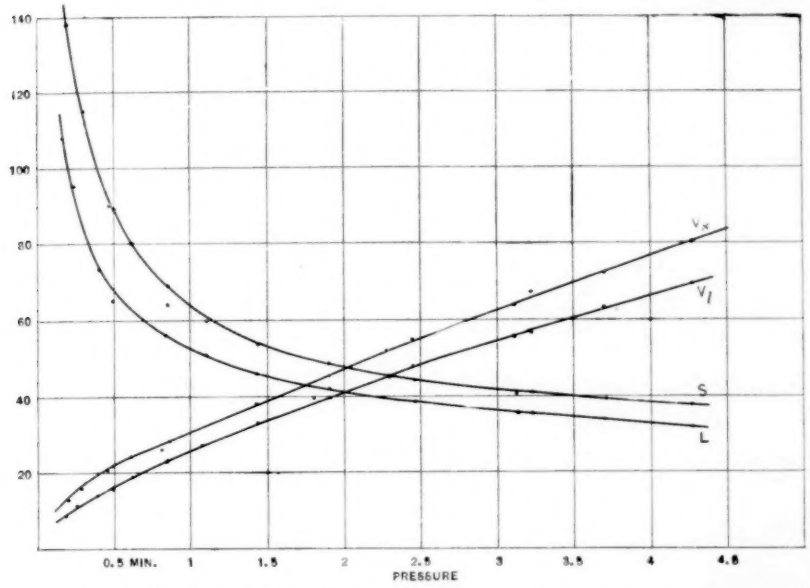

Fig. 9. V_x, V_l Potential Gradients. Ordinates = volts per cm. S, L, potential gradients ÷ pressure. Ordinates = V / p × 2.

for nitrogen may be assumed to hold to the same order of magnitude, the temperature corresponding to the data of Tables VI. and IX. may be as high as 450° absolute. The ratio of the values of Va would then be 470/450 575/583 = 1.03; and the ratio of efficiencies is .339/.323 = 1.05. The same change of 3 per cent. in the value of V/p at this pressure, 2.75 mm. (Fig. 9), corresponds to a pressure difference which would change the efficiency (Fig. 4) in the ratio 1.0375. The variation is thus in the right direction, and as nearly the right amount as could be reasonably expected under the experimental conditions.

Variation with Current.—In Fig. 10 are plotted the results obtained when the current is varied at constant pressure (Tables V. and IX.). The potential gradient curves show the variation in V

[1] Wied. Ann., 59, 244, 1896.

simply, since p is constant. a varies with the temperature, as in the case considered above, but in a less known way; so that calculations similar to the one given there are of little value. Using the slopes of the curves at the smallest current, where the temperature variation is small, results are obtained of about the same order of accuracy as those for the variation with temperature.

Total Radiation. — It was mentioned above that the energy transferred to the molecules during collisions is manifested partly by rise of temperature of the gas, and partly as radiation energy. When constant current is used, the potential gradient measures the supply of energy; and comparison of Figs. 5 and 9 readily shows that the radiation represents different fractions of the total energy at different pressures. Ångström[1] obtained an absolute measure of the radiation energy, and so found the values of these fractions. For nitrogen, they vary from 0.025 at 1.6 mm. to 0.075 at 0.12 mm. This variation doubtless has some effect on the efficiency ratio.

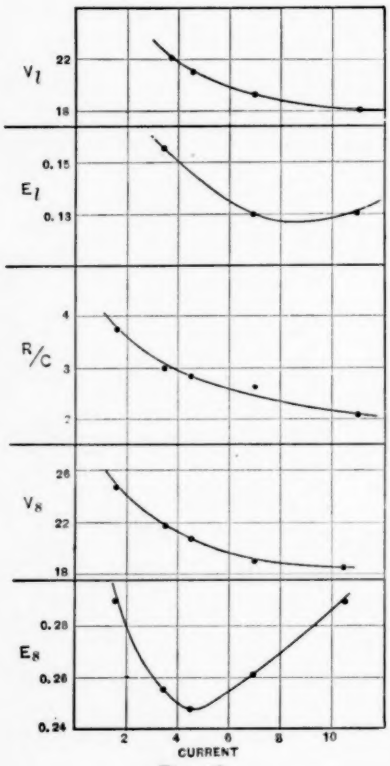

Fig. 10.

If however the pressure be kept constant and the current varied, this variation will not enter, and the total radiation should be proportional to the product of potential gradient and current strength; or potential gradient should vary with radiation divided by current strength. This latter quantity, R/C, is plotted in Fig. 10 for the results of Table V. and bears out this relation approximately, on comparing with the potential gradient curve just below. Temperature does not enter here, and so its variation causes no trouble.

[1] Wied. Ann., 48, 524, 1893.

This consideration, when applied to the total radiation per unit length of the two tubes, at the same pressure and current, shows that it should be in the same ratio as the potential gradients, and hence slightly less for the large tube. The results obtained by the photometer method, Table VIII. make it a little greater; but the difference is probably not greater than the experimental error.

Returning now to the consideration of the results shown by Figs. 4 and 9, it is evident that a comparison on the assumption of constant temperature is not legitimate. Heat is developed at nearly the same rate in the two tubes, and so the temperature must be much higher in the smaller one. If this be assigned a temperature 200° above the normal, the temperature in the large tube would be about 50° above. And this temperature difference would reduce the difference in efficiencies by perhaps forty per cent. of the larger. No conclusions can therefore be drawn as to the application of the theory to these results unless the temperatures of the gas within the tubes be measured. This the writer hopes to be able to do in the near future.

Summary.

The chief results of the investigation may be briefly summarized as follows:

1. The theory that the radiation from a vacuum tube is due to collisions between charged particles and neutral gas molecules, and that the proportion of high-frequency radiation increases with the energy of these collisions, is sufficient to account for the observed facts in all cases where the conditions are well enough known to warrant its application.

2. The luminous efficiency of vacuum-tube radiation by air at a pressure of 1 mm., under the conditions described, is approximately 0.20.

3. A considerable portion of this radiation is due to a single line, or narrow group of lines, at wave-length 4.75 μ.

It gives me pleasure, in concluding, to express my thanks to Professor Nichols, who suggested the investigation, and to Professor Merritt and Dr. Shearer, for their helpful suggestions as well as for the material facilities which they have placed at my disposal.

I am also greatly indebted to Mr. W. W. Coblentz for the privilege of using his apparatus, and also various results which he has obtained with it.

Physical Laboratory of Cornell University,
May, 1903.

ON ELECTRIFICATIONS PRODUCED BY GASES THAT HAVE BEEN EXPOSED TO ROENTGEN RAYS.

By John Zeleny.

1. Some years ago [1] I showed that when a piece of metal is surrounded by air which is partially ionized from exposure to Roentgen rays it acquires a negative charge, while the air itself becomes positively electrified. I showed how these charges could be augmented by increasing the surface of the metal that is exposed to the air and blowing the charged gas away from it. The explanation offered for the formation of the charges was based on the following considerations: It was found [2] that the oppositely charged ions formed in air by Roentgen rays have different migration velocities when in an electric field, and as the negative ions move the faster it was concluded that these ions are smaller in size than the positive ones. When the ions from a gas impinge on a bounding wall they give their charges to it, and so the concentration of the ions near the wall becomes less than it is in the body of the gas. On account of this difference of concentration the ions diffuse from the body of the gas to the wall, and as the negative ions diffuse the faster the bounding wall becomes charged negatively, while the excess of positive ions remaining temporarily in the gas gives it its positive charge.

More recently Villari [3] has carried on a large number of experiments with Roentgenized air blown through straight and coiled tubes, and through bundles of wires, gauzes and metal foil. Depending upon the conditions, the charges given to the metal in these cases were sometimes positive and sometimes negative. To explain his results, Villari has offered the hypotheses that when Roentgenized air passes by a metal surface the friction produces

[1] Philosophical Magazine, XLVI., 1898, p. 134.

[2] *Loc. cit.*, p. 132.

[3] Rendiconti della R. Accademia dei Lincei, IX., 1900. Physikalische Zeitschrift, II., p. 178.

a separation of the two electricities, that the positive electricity appears on the metal surface when the friction is above a certain amount while with less friction the metal becomes negative, and that the electricity of the opposite sign does not remain in the air but is used up in changing the Roentgenized air into ordinary air.

In the present paper, Villari's results and hypotheses will be considered in conjunction with some new experiments which have been performed, and it will be shown that all of the results find a general and simple explanation in ascribing their cause to the unequal size of the ions of opposite sign which are present in the gas.

2. For a clearer presentation of the subject, the experimental results will be deferred and a purely theoretical case will be considered first.

Suppose that AE (Fig. 1) represents a long, metal tube, through

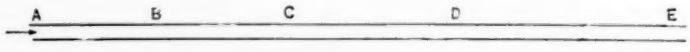

Fig. 1.

which a gas is made to flow in the direction from A to E. Suppose further that the gas entering at A contains an equal number of oppositely charged and uniformly distributed ions, of which the negative are the smaller and consequently the more mobile. As the gas passes along the tube, the ions gradually disappear from it because of the recombination of those of the opposite kind and because they discharge themselves when they come into contact with the walls of the tube. In the first part of the tube the negative ions, being the more mobile, diffuse the more rapidly to the walls of the tube, giving it a negative charge and leaving the gas itself charged positively. But as the gas passes along the tube the preponderance of the negative ions reaching the walls gradually gets smaller both on account of the relatively greater number of the positive ions remaining in the gas and because this excess of the positive gives rise to electrical forces that tend to drive them to the walls of the tube. There must be some point C, where the number of the two kinds of ions reaching the tube is the same. For all points to the right of C the positive ions get to the walls of the tube in excess, and some point E is finally reached where all of the ions have disappeared from the gas. The length of tube required

for this depends upon its cross-section and upon the velocity of the stream of air, increasing with both of these quantities. The gas issuing from the tube has lost its conductivity and is electrically neutral. The tube as a whole does not become charged, as the amount of positive electricity received must equal the amount of the negative, since the gas entering and leaving the tube is assumed to be without a charge.

3. The result of a number of modifications of the conditions can now be stated.

(*a*) If the tube is made shorter than AE and the velocity of the gas stream is maintained as before, the gas issuing from the tube will be charged positively while the tube itself will get a negative charge.

(*b*) If the tube is divided at B and the two parts insulated from each other the gas issuing from E will be neutral, the part BE will get a positive charge, and the part AB a negative charge.

(*c*) In the preceding case a piece DE of the tube may be removed such that the negative charge received by the part BC is just equal to the positive received by the part CD, so that the part BD as a whole will not become charged. The gas issuing from D will have a positive charge.

(*d*) If in the preceding case a piece of tube longer than DE is removed, the negative charge reaching the remainder will be greater than the positive and the tube will now become negatively charged while the gas issuing from it will be positive as before.

4. In any actual experiment where the air is exposed to the Roentgen rays in one vessel and from there blown into any tube under consideration, it is evident from the above that when it enters such a tube it will have a positive charge because the diffusion process has been going on before the air reaches the tube. This charge carried by the air into the tubes employed by Villari was left out of account by him although it seems that it played an important part in some of his experiments.

To make the matter more conclusive an experiment will be described which shows that such a positive charge does actually exist in the air as it enters the tube in an apparatus similar to that used by Villari. The main parts of the apparatus used for the ex-

periment are shown in Fig. 2. The air was sent from a large gas-bag through the tube A and through the plug of glass wool B into the large, cylindrical, tin can C, 15 by 28 cms. in size.

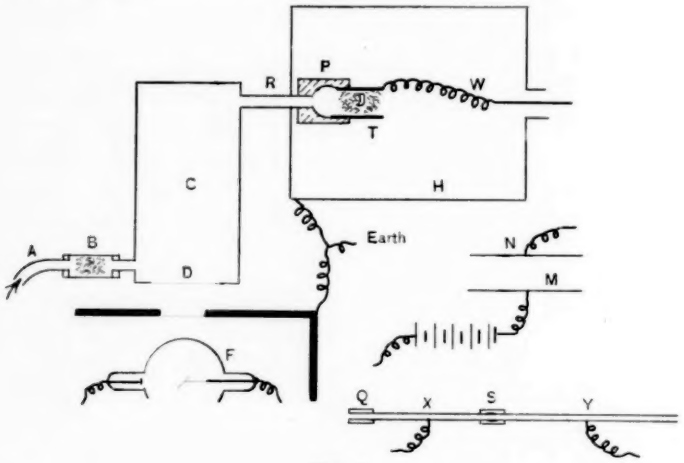

Fig. 2.

From C the air passed through the short brass tube R and out through a plug of glass wool contained in the short brass tube T, 2 cms. in diameter. The tube T was held and insulated by the paraffin tube P and was connected to one pair of the quadrants of an electrometer by the wire W. While in the can C the air was exposed to Roentgen rays which came from the bulb F and entered at the paper or aluminum window D. The rest of the apparatus was shielded from the rays and electrically protected by earthed metal screens.

It is well known [1] that the ions produced in gases by Roentgen rays are all removed by passing the gas through a plug of glass wool such as that contained in the tube T. So, by passing the gas through the above apparatus the electrometer indicates whether the gas coming out of the tube R is electrically neutral or whether it is charged with electricity of either sign.

On performing the experiment it was found that when air was used it always came out charged positively. The maximum amount of charge that can be collected and retained by T is dependent upon the intensity of the ionizing radiation, upon the

[1] J. J. Thomson and E. Rutherford, Philosophical Magazine, XLII., 1896, p. 393.

velocity of the gas stream, and upon the length of the insulation at P. Since the charge on the tube T is gradually dissipated by conduction through the ionized gas that approaches it through P, the charge reaches a fixed value when the rate of its acquisition is equal to the loss by conduction. For a given charge on T the leakage through the gas increases with the strength of the electric field between T and R and so is the smaller the greater their distance apart. It is on this account that the maximum charge retained by T increases with the length of the insulation at P.

In Villari's experiments an arrangement similar to that above was used, the point to which his experimental tubes were attached corresponding to that occupied by T, and hence it is concluded that the gas entering the tubes in his experiments was always charged positively. In any of his experiments, therefore, where the larger part of the ions diffused from the gas to the walls of the tubes, a resultant positive charge was necessarily obtained.

5. In one series of his experiments Villari found[1] that when he passed the Roentgenized air, coming from a can similar to C above, through a short tube this received a negative charge but when the air was passed through a long tube the charge acquired was positive. An explanation for this is readily seen from the considerations given in §§ 2 and 3, where it was shown (case d) why a short tube should acquire a negative charge, and (case b) why a long tube should become positively charged.

6. Villari found that air blown through a tube wound in a number of turns loses its conductivity more rapidly than if it goes through the same tube when straight. I performed some experiments with various lengths of rubber tubing interposed at the point R in the apparatus shown in Fig. 2. On blowing the Roentgenized air through these tubes both when straight and when coiled the charge received by T was positive in all cases.

P and T were then replaced by the two parallel plates M and N shown in the figure. M was kept at a potential of ten volts by a battery and N was connected to the electrometer. The conductivity of the gas issuing from R was measured by means of these plates.

In one case when the rubber tube placed at R was 90 cms. long and 6 mm. internal diameter, the charges received in one minute by

[1] Rend. Lincei, 1900, p. 13.

the plate N when the tube was arranged in one large loop were + 65 electrometer divisions with M positive and − 45 electrometer divisions with M negative. When the same tube was wound in eight small loops the deflections obtained were + 13 divisions with M positive and − 4 divisions with M negative.

With a straight tube 19 cms. long the deflections averaged 166 divisions for the same time, while when the tube was made 230 cms. long the deflections were just barely appreciable. It is noticed from the numbers first given that there was an excess of positive ions coming from the tubes.

The greater loss of conductivity in passing through a tube wound in several loops is to be explained in the following way. When a tube is nearly straight the motion of the gas through it is steady, with the various portions of the gas moving in lines parallel to the axis of the tube. If, however, a tube is much curved, the conditions for steady motion no longer obtain and the motion is turbulent. With steady motion the ions from the center of the tube can get to the walls but slowly by diffusion. With turbulent motion, however, the ions are carried by the eddies from the center to the walls and so are discharged more rapidly.

Dorn [1] has offered an ingenious explanation for the positive charge that a tube takes on when it is wound in a coil and Roentgenized air is passed through it. He suggests that the ions maintain their direction in turning a corner on account of their relatively large mass, and as the mass of the positive ions is the larger, that more of these strike against and are discharged to the walls. But a straight tube if long enough also receives a positive charge under the same conditions and Dorn's explanation does not hold for such a case.

It seems rather that the charge observed in all such cases is that which enters the tube with the gas, as explained in § 4. This charge accumulates on the tube, if this is long enough for the ions to discharge to its walls while the gas is passing through. A shorter tube suffices if it is coiled since the ions discharge more rapidly in it, owing to the turbulent motion.

7. Villari found that the conductivity of the Roentgenized air is much diminished by passing it through rolls of foil and gauzes, and through bundles of wires and strips of metal. The division of the

[1] Physikalische Zeitschrift, II., p. 238.

air stream into small channels, thereby increasing the discharging surface and diminishing the distance that the ions have to go to diffuse to the walls, and this aided in some cases by the turbulent motion produced, at once accounts for the diminished number of ions that gets through any such an arrangement.

He also finds that the rolls of gauzes become charged positively if they are long and closely pressed, and that they acquire a negative charge if they are short and loose.

Likewise, coarse copper turnings become positive when they are pressed compactly in a tube and become negative if they are put in loosely.

In all cases where the air, when it leaves the apparatus, has passed through only the first stages of the diffusion process explained in § 2, the metal parts receive a negative charge, while if the diffusion of the ions has already approached the last stages then these metal parts get the positive charge which enters the tube from the ionizing can (see § 4).

8. Villari sums up his results with the statement that all metals become charged positively when the air rubs over them with a certain amount of energy, and that they become charged negatively when the air rubs with less energy.

This statement fits all of his experiments but it will not apply to all cases. For instance, in one of my experiments the glass wool in the tube T of Fig. 2 was replaced by a roll of brass foil. On blowing very gently through this it acquired a positive charge, while for a more rapid stream it became charged negatively. Here with more energetic friction the charge was negative which is the opposite of what Villari's statement requires. The result is explained by considering that with the very slow motion the ions have so nearly all diffused out of the gas for the case to come under *d*, § 3, while for a faster stream case *b* applies.

In agreement with the ideas presented in § 2 and § 3, it was found that the gas coming out of such rolls of foil is invariably charged positively, no matter whether the foil itself is getting a positive or a negative charge.

9. Another experiment will be cited as evidence against the idea that the sign of the charge received by a tube depends upon the

amount of friction of the air passing through it. The parts P and T of the apparatus shown in Fig. 2 were replaced by the arrangement $QXSY$ shown at the side of the figure.

Here X and Y are pieces of brass tubing 2.5 mm. in diameter and 14 and 42 cms. long, respectively. The ebonite piece S insulates X from Y and the whole is attached to and insulated from R by the ebonite piece Q. One of the tubes was always connected to the electrometer and the other to earth. When the Roentgenized air was sent through these tubes with a velocity within a certain range, X received a negative charge and Y received a positive one. As the velocity of the gas was the same through both tubes and these were of the same diameter the air friction must have been the same in both; and yet the charges received by them were of opposite sign.

An explanation is at hand if we consider that in the tube X the negative ions are still diffusing to the walls in excess while in Y the positive ones predominate, the two tubes corresponding to the parts BC and CD, respectively, of the tube shown in Fig. 1.

10. One of the strongest confirmations of the explanation given for the various results that have been considered, *i. e.*, that the observed effects are due to the unequal size of the ions of the two kinds, is found in the experiments performed with carbonic acid saturated with water vapor. When I determined the ratio of the velocities of the two ions produced in gases by Roentgen rays,[1] the effect of the presence of water vapor in the gas upon the ionic velocities was not known. The results obtained showed that in air the negative ions move the faster while in the undried carbonic acid that was used the velocities of the two ions were practically equal. So at that time,[2] in connection with the experiments already referred to in § 1 where the charge carried by air after coming through a roll of metal foil was found to be quite strongly positive, the same experiment was performed with the carbonic acid. With this gas the positive charge obtained was so very small, comparatively, that the experiment was there cited as strong evidence that the cause of the charges is the inequality in the rates of diffusion of the two ions.

[1] Philosophical Magazine, XLVI., 1898, p. 120.

[2] *Loc. cit.*, p. 136.

More recently[1] I have determined the absolute velocities of the two ions separately in a number of gases both when dry and when saturated with water vapor. It was found that the velocity of the positive ions in carbonic acid that is saturated with water vapor is greater than that of the negative, while in dry and moist air and in dry carbonic acid the negative ions have the greater velocity. If the inequality in the size of the two ions is the cause of the electrical charges under consideration then the charge to be obtained in any case with carbonic acid that is saturated with water vapor should be opposite in sign to that obtained with air or dry carbonic acid.

For this reason Dorn suggested to Villari[2] that he repeat some of his experiments using moist carbonic acid instead of air. In doing so Villari obtained in two cases a charge of the same sign as with air, although it was much smaller. In a third case the carbonic acid gave a slight and uncertain charge where air gave a strong positive charge.

The results of the experiments that I performed, which will now be considered, are not in agreement with those of Villari.

The carbonic acid used was taken from a cylinder of commercial, liquid carbonic acid. For saturating the gas with water vapor it was first bubbled through the water in a large bottle containing a large number of glass rods, and then passed through a long horizontal tube half filled with water. To dry the gas it was passed through a tube of calcium chloride.

An apparatus equivalent to that shown in Fig. 2 was used. The brass tube T was somewhat longer, contained glass wool as before, and was connected to the electrometer. It was joined to and insulated from R by a piece of ebonite tubing. At its other end it carried a second piece of ebonite tubing from which a rubber tube led to a second large gas-bag. The gas could thus be made to pass from the first gas-bag through the drying or moistening apparatus, through the can C and the glass wool in T, and on into the second gas-bag. By interchanging the tubes attached to the two bags the gas could be used again, being more thoroughly dried or moistened

[1] Phil. Trans. of the Royal Soc. of London, 195, 1900.

[2] Physikalische Zeitschrift, II., p. 360.

by a second passage through the proper substance. It will be noticed that in this experiment, as explained in § 4, the charge gathered by the glass wool is that left in the gas because of the faster diffusion of the ions of the opposite sign to the walls of the vessel C and the connecting tube R.

The following results are given as typical of a number of experiments performed on different days. The readings are given with their proper signs in terms of divisions of the electrometer scale, the time that the action had gone on being indicated in each case.

Time.	Dry Air.	Moist Air.	Dry Carbonic Acid.	Moist Carbonic Acid.
30 seconds.	+ 59	+ 12	+ 29	− 19
60 "	+130	+ 25	+ 62	− 39
90 "	+196	+ 36	+ 93	− 57
120 "	+256	+ 46	+126	− 73

It is seen that the charge received by the glass wool was positive in all cases except when moist carbonic acid was used, in which case the charge was negative. As stated, in this last gas the positive ions are the smaller while in all of the other cases they are the larger.

Although the same weights of 100 lbs. were placed on the gas-bag in each case, the two results for either gas are not numerically comparable as the velocity of the gas stream was not the same in the two cases, more resistance being offered to the gas in bubbling through the water than in passing through the drying tube.

The above experiments were varied by replacing the glass wool with a loose roll of brass foil and dispensing with the second gas bag, so that the gas issued directly into the open air. The brass roll was connected to the electrometer and when dry or moist air or dry carbonic acid was passed through it from the ionizing can the charge received by the roll was negative, but when moist carbonic acid was used the charge received was positive. These results are likewise explained by the faster diffusion of the smaller ions in each case to the walls of the roll of brass foil. It should be remarked that when dry carbonic acid was used after the moist had been employed a considerable quantity of the gas had to be passed

through the apparatus before the moisture was all taken up from its walls, and likewise in using the moist gas after the dry it seemed at first to lose some of its moisture to the tubes and apparatus, so that the reversal of the charge obtained did not begin until the gas had been flowing for some time. This last would indicate that the gas has to be quite near to the saturation point before the negative ions become larger than the positive. I cannot account for Villari's failure to get the reversal of effects with the moist carbonic acid otherwise than by supposing that his gas did not contain enough water vapor to produce the reversal in the relative dimensions of the two ions.

11. The unequal diffusion of the two kinds of ions has been given as the general cause of the electrifications that have been considered. There are doubtless other causes which under special conditions produce or modify to a greater or less extent the charges that are obtained. It has been known for some time that when two different metals are used, an ionized gas will produce charges upon them by the reduction of the contact difference of potential.

It is thought possible that the amount of the surface layer of gas on a metal may influence the charge that is received by ionic diffusion, and that this charge may on this account differ in amount for different metals.

In a recent paper[1] J. C. McLennan and E. F. Burton consider radioactivity as a cause for the charging of metals.

12. One of the hypotheses, already mentioned in §1, which Villari offers to explain some of his experiments,[2] is that when Roentgenized air is changed into ordinary air this is done at the expense of an electric charge which vanishes. The generally accepted view of the process whereby a gas loses its conductivity is that the ions which give the gas this conductivity disappear from it either because of the recombination of those carrying opposite charges or because of their discharging themselves against some surface whither they have been carried either by electrical or mechanical forces. Unless opposed by facts, this view must be preferred as it evidently

[1] University of Toronto Studies, 1903.

[2] Rend. della R. Accademia di Bologna, 1900.
Physikalische Zeitschrift, II., p. 180, § 16.

does not require any such a strange assumption as the destruction of an electric charge.

One of the experiments upon which Villari bases his hypothesis will now be considered. In this he took a tube of paraffin and gave its inner surface a static charge of either sign. On blowing Roentgenized air through the tube he found that the air lost its conductivity and the paraffin tube its charge. If I understand him rightly, he reasons that since the charge lost by the paraffin was not blown out of the tube with the air and since the air lost its conductivity in passing through the tube that therefore the charge had been used up in transforming the Roentgenized air into ordinary air.

But if the charge is not carried out of the tube in the direction in which the air is moving, there is still the possibility that the ions in the gas may carry it out backwards against the air stream. That this is what actually takes place in such a case was proven by means

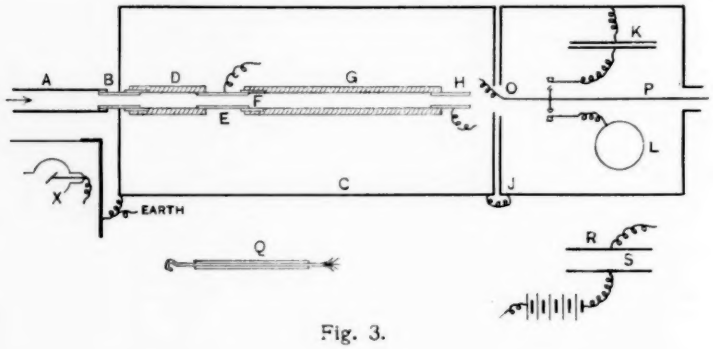

Fig. 3.

of the apparatus shown in Fig. 3. Here G is an ebonite tube 17.5 cms. long, having an internal diameter of 1.6 cms. It carries at one end the short brass tube H and at the other end the brass collar F, by means of which it can be readily attached to the brass tube E without the development of any static charges. E is carried and insulated by the ebonite tube D. Air, blown from a gas-bag or bellows into the cardboard tube A, is there exposed to the Roentgen rays from X, and then passes through the tubes D and G. The tubes E and H are connected, one to earth and the other by means of the wire OP to a quadrant electrometer. All necessary parts of the apparatus are screened from the rays and from outside electrostatic influences.

G is the non-conducting tube from the inner surface of which a static charge is made to disappear by blowing Roentgenized air through the tube, and it is the object of the experiment to find out what becomes of this charge.

The metal rod Q, covered with an ebonite tube and carrying a metal brush at one end, was used for charging G. The rod was connected to one pole of a small static machine and the tube G was slid over it so as to receive the discharge from the points of the metal brush on its inner surface.

The conductivity of the gas issuing from the tube H could be tested by placing at O the two plates R and S, one of which was connected to a battery of cells and the other to the electrometer. When the inner surface of G was given a charge of either sign, the Roentgenized air that entered the tube was found to lose its conductivity by passing through it and did not recover it completely until G had lost all of its charge. In the meantime E was found to receive a charge of the same sign as that given to G while H received a charge of the opposite sign. Both of these charges at first sent the electrometer deflection off the scale rapidly, but the charge going to E was much the larger of the two. It will be shown that the charge received by E was equal in amount to the charge originally given to the tube G and, as has been said, it was of the same sign. The charge received by H results from the change in potential of H owing to the removal of the charge from G. This last fact was proved in the following manner: In order that the total charge received by H might not deflect the electrometer off the scale, the sensibility of the instrument was diminished by joining the wire leading to it to an induction plate situated above one of the quadrants, instead of connecting it directly to the quadrants as was done before. If during the charging of G the tube H was kept insulated and when G was introduced into the apparatus H was connected to the insulated induction plate, the electrometer showed a deflection of the same sign as that of the charge on G owing to the charge induced on H by that on G. That is, H had a bound charge of the opposite sign to that on G and the equal free charge of the same sign as G produced the deflection on the electrometer.

When the Roentgenized air was now blown through the tubes of the apparatus, the electrometer deflection gradually diminished and the reading on the scale finally became stationary at practically the zero position. Hence during the experiment H actually received no appreciable charge, but the removal of the charge from G simply allowed the two induced charges to reunite. This shows that in the former experiments, where H had been brought to zero potential before the Roentgenized air was sent through the apparatus, the charge of the opposite sign to that of G that was indicated by the electrometer was due to nothing more than the liberation, by the discharge of G, of the induced charge that H possessed.

The following procedure was pursued to show that the charge received by E was equal to the charge first given to G. A capacity K of .02 microfarad and a hollow cylinder L were connected by the keys indicated in the figure to the wire OP leading to the electrometer. The large capacity was necessary to keep the deflections on the scale. The cylinder was used to measure the charge initially given to G by the static machine. An opening in the box J above the axis of L was uncovered and the charged tube G was carefully lowered into the cylinder. The deflection of the electrometer indicated the total charge on the tube. G was then placed into its position in the apparatus, and E was connected to the electrometer and H to earth. The Roentgenized air was next passed through the apparatus and the electrometer deflection noted from time to time. As an example of the results obtained the following readings, among the best obtained, are given.

When the tube G was lowered into the cylinder L the deflection obtained for its charge was + 132 divisions. With G in the apparatus and E connected to the electrometer the readings obtained, after the gas had been passing for the times indicated, were:

+	20	divisions in	1	minute.
+	56	" "	3	minutes.
+	92	" "	5	"
+	123	" "	7	"
+	129	" "	8	"
+	129	" "	9	"
+	128	" "	10	"

The tube G was now removed and when tested was found to be discharged.

It is seen that the charge received by E was practically equal to the charge lost by G. These is therefore no annihilation of electricity here. The charge is, as it were, simply transferred by the conducting gas from G, against the gas stream, to E. The gas loses its conductivity in the same way as in all cases where it passes through an electric field. As the ionized gas comes out of E it enters the electric field existing between the charges on G and E. The ions with charges opposite to that of G travel to G and gradually neutralize its charge while those of the same sign as G go to E and give it the charge that is indicated by the electrometer. The gas loses its conductivity in passing through the tube because it loses its ions.

It should be noted that the charges given to G in these experiments were large. In fact if the tube was held near to a conductor there was an audible discharge from its outer surface. On this account the greatest care had to be taken to prevent leakage. The effect of the charges obtained by diffusion, described previously, was comparatively so small as to be negligible. In the last experiment certain induced charges had to be guarded against and taken into account, into the details of which it is not thought necessary to enter.

13. *Summary.* — Roentgenized air which has been blown through tubes or rolls of metal foil is charged positively, so long as it retains any conductivity at all. The same tube or roll of metal foil through which Roentgenized air is being blown may receive a positive or a negative charge, depending upon the velocity of the stream of air. Under proper conditions the first part of a tube may receive a negative charge while the more distant part of the same tube receives a positive charge. When moist carbonic acid is used the charges developed in the gas and on the metal are the opposite to those obtained under like conditions with dry or moist air or dry carbonic acid.

A general explanation for all of these results is found in ascribing their cause to the unequal velocities of diffusion of the two kinds of ions in the gas. Villari's hypothesis that the rubbing of the Roentgenized gas against a surface produces a separation of the two electricities, of which either may appear on the metal depending

upon the amount of friction, is not sufficient to explain all of the facts and is therefore untenable.

When Roentgenized air is blown through an ebonite tube having a static charge on its inner surface, this charge is eventually all carried by the ions to the nearest conductor of large capacity against the gas stream.

Villari's assumption that in such a case the charge vanishes, and that in general when Roentgenized air is changed into ordinary air this is done at the expense of an electric charge, is not upheld.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA.

ON THE RADIANT EFFICIENCY OF THE NERNST LAMP.

By L. R. Ingersoll.

ALTHOUGH many tests of the commercial efficiency of the Nernst lamp have been made, there seems to be little available data as to its absolute luminous efficiency, besides the statement in Drude's *Lehrbuch der Optik*[1] placing it at 12 per cent. The object of the present work, therefore, was a determination of the efficiency of the Nernst glower under varying conditions, and an estimate of its temperature from the results. Incidentally, also, it was hoped that the work would give an indication of the accuracy which might be expected of the method employed, which was, in its essentials, the comparatively new one due to Ångstrom[2] and used by him in connection with his determination of the mechanical equivalent of the unit of light.

It is a fact of only too common observation, that different observers, in determining the efficiency of the same kind of light source, arrive at very different results. The discrepancy is doubtless due only to difference in the methods employed, but is of such magnitude as cause one to question results obtained in almost any of the older ways. Each of the two well-known methods of determining efficiency is open to objection. That of using a cell of absorbent solution to free the light from infra-red rays is now generally accepted as unreliable, and, as shown by E. L. Nichols,[3] may lead to widely varying results. That employed by Langley and others, of integrating the energy curves obtained with a bolometer, is unnecessarily complicated when one wishes merely the ratio of the total and visible radiations. Furthermore, strong selective

[1] Drude, Lehrbuch der Optik, p. 447.

[2] Astrophys. Jour., XV., 223. April 1902.

[3] Paper before the A. A. A. S., Washington, 1902. Also Phys. Rev., 17, p. 267.

absorption in the infra-red by different elements of the apparatus renders it sometimes difficult of application.

Ångstrom's method, however, is comparatively free from the above objections and is moreover susceptible of corrections for selective absorption and reflection which are simple and easy of application. In brief the scheme is to disperse the light and screen off all but the visible portion of the spectrum. This part is again combined into white light by a cylindrical lens and balanced photometrically against the total radiation from another exactly similar source. The energy of the two radiations can then be compared by replacing the photometer screen with a thermopile or bolometer and observing the galvanometer deflections for each light. The method is peculiarly well adapted to the Nernst lamp as the great brilliancy of the glower makes possible the use of a narrow slit without too great a sacrifice in intensity.

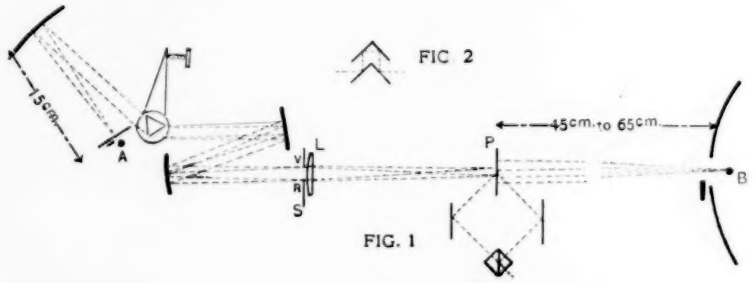

FIG. 1

The general arrangement and dimensions of the apparatus may be seen from the diagram. The two glowers A and B were burned freely in air, without heaters, on a 110-volt alternating current. A voltmeter and wattmeter across the terminals of glower B and a rheostat in series with both lamps enabled one to keep the voltage or power consumption constant at any desired value. As a small difference in voltage makes a considerable difference in the brilliancy of a glower, it was necessary to allow for variations in the pressure furnished by the dynamo, by frequent adjustments of the rheostat. The slit, mirrors, prism and screen were fastened to an iron plate for the sake of rigidity. To secure sufficient intensity, the mirrors were of rather short focus, but as only a small central area of each was used, the aberration error was small. The setting of the screen

S to cut off the infra-red was made by focusing sunlight on the slit and then with the aid of an eyepiece, setting the screen so that its edge just coincided with the A line $\lambda .76\,\mu$ and clamping it there, the final adjustment being completed by rotating the prism with the tangent screw. This way of adjusting has the advantage of enabling one to secure as fine an adjustment, and to work as near the position of minimum deviation, as desirable. A second screen, approximately placed, cut off the ultra-violet also. Just behind the screens was placed the cylindrical lens L of 14 cm. focal length which recombined the visible spectrum into a strip of white light 1 cm. wide and 3 cm. long.

To compare the energy of the two radiations the photometer head was replaced by a Rubens thermopile connected with a small Thomson galvanometer, whose normal sensibility of $\sigma .10^{-9}$ could be reduced by inserting resistance to about one fourth of that, when measuring the unobstructed radiation from the lamp B. The galvanometer zero was obtained in each case by closing a small shutter in front of the slit or before the second glower B.

In many of the first experiments a small water cell was mounted directly in front of the screen S to absorb any stray infra-red rays due to impurity of the spectrum, but as careful tests failed to discover any appreciable impurity, its use was later abandoned.

The Nernst glower A was mounted so as to allow of adjustment parallel to the slit and was fastened at its ends to prevent its being warped out of position by heating. Glower B was mounted on a carriage sliding on an optical bench about a meter long, and was partly surrounded by a covering with a small opening facing the photometer screen. The photometer head — a type of Lummer-Brodhun — also mounted on the optical bench, was fixed at one position, the photometric balance being made by moving the lamp B with a lever arm. Successive settings of the latter generally varied by less than 2 per cent. and were made in sets of 18 usually, half with the head reversed, and at three different parts of the strip of white light. The proper direction in which to point the thermopile was found by mounting it on a horizontal divided circle and noting the angle for which the galvanometer deflection was a maximum.

Corrections were made for the following sources of error:

1. Selective reflection by the three silvered mirrors.

2. Selective absorption by the glass prism and lens, or by the water cell (where used).

3. Inaccuracy in setting of the screen at the *A* line.

4. Non-uniformity of the strip of white light.

5. Disturbing effect of the wattmeter across glower *B*.

6. Radiation from the shutters before the lamps.

The general scheme of obtaining a correction was to exaggerate the error and note the effect on the final result. A proportional correction could then be deduced. Thus in testing the effect of selective reflection at the three silvered mirrors, a set of four plane silver surfaces (Fig. 2) was mounted just behind the cylindrical lens. The effect of the four additional reflections was to increase the observed efficiency by nearly 7 per cent. of its value (an actual increase of 0.28 per cent. for an efficiency of 4.2 per cent.). The proportional correction of 5 per cent. for the three mirror surfaces is in good agreement with a theoretical correction of 4 per cent. which was roughly calculated from data on the reflecting power of silver for various wave-lengths, given by Hagen and Rubens.[1]

In the same way, by introducing an extra thickness of glass in the path of the luminous radiation, the correction for the selective absorption by the prism and lens was found to be negligible. The effect of the water cell, when used, was also allowed for, the correction being slightly less than that for silver, and opposite in sign. It is evident of course that general absorption, which is non-selective in character, cannot introduce any error.

The accuracy with which the screen to cut off the infra-red could be set at the *A* line was estimated at about the width of the line itself. This would lead to an uncertainty of about 3 per cent. of the result — the neighborhood of the *A* line being of course the most effective part of the visible spectrum as regards energy. However the setting at the beginning of any series of experiments, was supplemented by an observation of the position of the screen at the end, and if any apparent shift or displacement of the line was observed, a correction calculated for half this error was applied, so

[1] Zeit. fur Instr., 19, p. 293. 1899.

the probable error from this source is something less than 2 per cent.

The uniformity of the strip of white light formed by the cylindrical lens was tested photometrically at twenty different points of the area occupied by the thermopile. Diminishing intensity at the ends tended to make the efficiency readings too small and required a correction of 2.5 per cent. (actually 0.10 per cent. for an efficiency of 4 per cent.).

The effect of the measuring instruments connected with glower *B* was to run this lamp at a slightly lower voltage than the other and hence destroy the perfect similarity of sources presupposed by this method. Tests made by connecting proportional resistances across glower *A* showed that the effect of this source of error was to lower the observed efficiency 2.5 per cent. Radiation from the shutters and consequent displacement of the galvanometer zero required a correction of something less than 1 per cent. Proportionality corrections were applied to all galvanometer readings. In no case, however, did the sum of all corrections amount to more than 10 per cent. of the result.

The experiments indicate that the principal factors in determining the efficiency of any glower are its age and the power it consumes. A table with some typical results is given on the following page.

Glowers are by no means uniform. New glowers show an efficiency of from 4.35 per cent. ($\pm$.10 per cent.) to 4.70 per cent. ($\pm$.10 per cent.) — mean 4.61 per cent. The efficiency falls rapidly for about the first twenty hours, decreasing to 4.3 per cent. and varies only slowly after this. Tests on glowers of 40 hours age and upwards gave efficiencies of from 3.9 per cent. to 4.4 per cent.— mean 4.17 per cent. Some very old glowers gave results of only 3.6 per cent. It is noticeable that after a glower has been burned upwards of twenty hours it develops a marked crystalline appearance and it is probable that the fall in efficiency is due to the greater radiating surface and consequent lower temperature afforded by the crystalline structure.

The above figures are all for 110-volt glowers furnished by the Nernst Lamp Company, of Pittsburg, and consuming 89 watts power. For every watt above 89 within narrow limits the efficiency increases

Age of Glower.	Power Consumed.	Observed Efficiency.	Total Correction.	Corrected Efficiency.
New.	89 watts.	4.85%	−.15%	4.70%
"	89 "	4.61	.00	4.61
5 hours.	89 "	4.23	+.13	4.36
New.	88 "	4.44	−.09	4.35
18 hours.	88 "	4.15	−.10	4.05
42 "	88 "	4.10	−.09	4.01
New.	91 "	4.84	−.17	4.67
8 hours.	90 "	4.67	−.11	4.56
21 "	90.5 "	4.48	−.14	4.34
Over 40 hours.	89 "	4.05	+.23	4.28
" " "	" "	4.02	+.23	4.25
" " "	" "	4.03	+.23	4.26
" " "	" "	3.99	+.23	4.22
" " "	" "	4.06	+.24	4.30
" " "	" "	3.89	+.17	4.06
" " "	" "	4.10	+.16	4.26
" " "	" "	3.85	+.16	4.01
" " "	" "	3.80	+.30	4.10
" " "	" "	3.93	−.04	3.89
" " "	" "	4.20	−.05	4.15
Very old.	" "	3.63	−.05	3.58
" "	" "	3.63	−.01	3.62

0.06 per cent., and below, vice versa. In the above table the correction contains a reduction to this uniform condition.

The energy curve plotted from Wien's law which gives the efficiency 4.17 per cent., *i. e.*, which gives the ratio visible energy / total energy = .0417, is that corresponding to the temperature 2,360° C. absolute. This then would be the temperature of the Nernst glower considering it as a black body. It will be noted that it lies well within the limits, 2,200° to 2,450°, given by Lummer and Pringsheim,[1] their estimate being based on the position of λ max in the energy curve.

Another check on the above work as well as an indication of the extent to which the Nernst glower follows the laws of a black body is afforded by noting the increase of efficiency with power consumed. If we assume a temperature of 2,360° C. for an ordinary 110-volt glower taking 89 watts with an efficiency of 4.17 per cent., the same glower taking 95 watts, will have an efficiency of 4.53 per cent. indicating a rise in temperature of about 40 degrees. Now if the

[1] Verh. deutsche Phys. Ges., 190, 13, p 36–46.

glower acts as a black body the total radiation will vary as the fourth power of the temperature, or the ratio of the two radiations will be $2{,}400^4/2{,}360^4 = 1.069$. In reality this ratio will be very approximately $95/89 = 1.067$, supposing the convection and other losses to be a constant fraction of the whole. Of course the closeness of this agreement is only accidental, the probable error being much greater than the difference between these figures.

I wish to acknowledge my indebtedness to Prof. C. E. Mendenhall and Prof. Snow, of the physics department, for material assistance as well as many valuable suggestions in connection with this work.[1]

PHYSICAL LABORATORY, UNIVERSITY OF WISCONSIN,
July 21, 1903.

[1] Mr. L. W. Hartman in a recent paper on the luminous radiation from the Nernst glower (PHYS. REV., 17, p. 65), which has appeared since the above was written, notes that old glowers seem to show an increase in light intensity instead of a decrease as the above work would indicate. The discrepancy is probably due to the fact that his results were obtained while keeping the current rather than the power consumption, constant. As the resistance of a glower increases with age a constant current means an increase of power consumption and hence a greater emission.

Mr. Marshall Hanks of the Nernst Lamp Co. informs me that the commercial efficiency curves show a characteristic drop, but that an effort is at present being made to age the glowers before sending them out, hence future tests may not show this effect.

QUATERNIONS IN ELECTRICAL CALCULATIONS.

BY OLIN J. FERGUSON.

PRELIMINARY.

MAXWELL, in his works, uses quaternions in their most general form. This paper is intended to analyze the quaternion in two special cases, and for the benefit of electrical students who are not acquainted with this form of expression a few preparatory remarks are given.

Conceive of three operators i, j, k, $\perp$ each to each (see Fig. 1). Let the operation of any one of these upon another signify the revo-

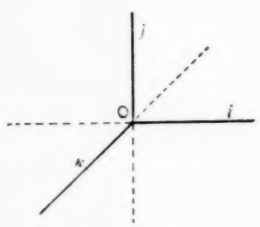

Fig. 1.

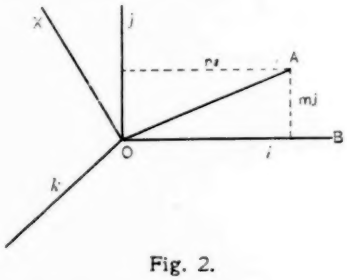

Fig. 2.

lution of the one operated upon through a right angle toward the left of the observer when he is standing upon the operator, looking toward the one operated upon.

That is,

$$ij = k; \qquad jk = i; \qquad ki = j;$$

$$ji = -k; \quad kj = -i; \quad ik = -j.$$

Therefore,

$$i(ij) = -j, \text{ or } i^2 = -1.$$

Similarly,

$$k^2 = j^2 = i^2 = -1.$$

If we let i be the unit vector along the i-axis, any vector coinciding with this axis in direction will be represented by a numerical

multiple of i. For example, ai, bi, etc. Making the same assumption for j and k, we see that a vector in the plane of the ij-axis may be expressed by the sum of i- and j-vectors (see Fig. 2).

$$OA = ni + mj,$$

or, if OA is of length a units,

$$OA = (a \cos AOB)i + (a \sin AOB)j.$$

In general, any vector through O, say OX of length x units, will be expressed by

$$OX = (x \cos \beta)i + (x \cos \gamma)j + (x \cos \delta)\, k,$$

where β, γ, δ are the angles between OX and the i-, j-, k-axes respectively.

We will now assume that any real quantity when acted upon by i will be revolved to lie along the i-axis. Then iA is a vector along the i-axis. Therefore,

$$iA = ai.$$

But from previous conditions,

$$i(iA) = -A.$$

That is, two operations by i revolves A through 180°, or each operation revolves it through 90°. Therefore, A is perpendicular to i. Similarly, A is perpendicular to both j and k, and we have four axes at right angles each to each. A vector having for its components certain projections upon each of these four axes will have the length expressed by the square root of the sum of the squares of these resolved parts.

As for terminology, a real part of a quantity is called a "scalar," and is measured along the real axis. The length of a vector is its "tensor," and is expressed TX (read "tensor of X").

Impedance.

Z = impedance.

Assume a vector E, of length e (that is, TE = e), rotating in the plane ij with the angular velocity ω. Let t represent the time elapsed since the start from the i-axis. The equation to represent this vector is,

$$E = \mathrm{e}\,(i \cos \omega t + j \sin \omega t).$$

The part $i \cos \omega t + j \sin \omega t$ is called the "versor."

Let E represent a harmonically varying E.M.F.

Let, also,

$$I = \mathrm{i}\,(i \cos \overline{\omega t - \theta} + j \sin \overline{\omega t - \theta})$$

represent the resulting current, with a difference of phase θ.

$$TI = \mathrm{i}.$$

$$E = ZI.$$

By definition, E/I is the operator which, acting upon I, will produce E.

$$(E/I)I = E = ZI.$$

Therefore,

$$E/I = Z.$$

But,

$$I = \mathrm{i}\,(i \cos \overline{\omega t - \theta} + j \sin \overline{\omega t - \theta}).$$

Multiply the above value of I by the assumed quantity,

$$\frac{\mathrm{e}}{\mathrm{i}}(\cos \theta + k \sin \theta),$$

and we will get

$$\mathrm{e}\,(i \cos \overline{\omega t - \theta} \cos \theta + j \sin \overline{\omega t - \theta} \cos \theta$$
$$+ ki \sin \theta \cos \overline{\omega t - \theta} + kj \sin \theta \sin \overline{\omega t - \theta}).$$

This reduces to

$$\mathrm{e}\,(i \cos \overline{\omega t - \theta} \cos \theta - i \sin \overline{\omega t - \theta} \sin \theta$$
$$+ j \sin \theta \cos \overline{\omega t - \theta} + j \cos \theta \sin \overline{\omega t - \theta}),$$

which equals

$$\mathrm{e}\,(i \cos \omega t + j \sin \omega t) = E.$$

Therefore,

$$E/I = Z = \frac{\mathrm{e}}{\mathrm{i}}(\cos \theta + k \sin \theta).$$

This kind of a quantity is called a quaternion and consists of the product of a tensor, $\frac{\mathrm{e}}{\mathrm{i}}$ and a versor, $\cos \theta + k \sin \theta$, or may

be looked upon as the sum of a scalar, $\frac{e}{i}\cos\theta$, and a vector, $\frac{e}{i}\,k\sin\theta$.

These parts have the following interpretation:

$$\text{Scalar part} = \frac{e}{i}\cos\theta = r.$$

$$\text{Vector part} = k\frac{e}{i}\sin\theta = kx.$$

$$\text{Tensor part} = \frac{e}{i} = z.$$

$$\text{Versor part} = \cos\theta + k\sin\theta = k^{\theta/\pi/2} = k^{2\theta/\pi}.$$

The quantities, r, x, z, have the ordinary interpretation and

$$z = \sqrt{[r^2 + (\omega L - 1/\omega C)^2]}.$$

The versor is the "turning" operator. $\cos\theta + k\sin\theta$ turns I through the angle θ and is therefore only a part of k, which would turn it through one right angle. The relation between the two is expressed above.

POWER.

$$E = e(i\cos\omega t + j\sin\omega t).$$

$$I = i(i\cos\overline{\omega t - \theta} + j\sin\overline{\omega t - \theta}).$$

$$EI = \text{power} = P.$$

$$P = ei(i^2\cos\omega t\cos\overline{\omega t - \theta} + ij\sin\omega t\cos\overline{\omega t - \theta} + ji\cos\omega t\sin\overline{\omega t - \theta} + j^2\sin\omega t\sin\overline{\omega t - \theta}).$$

$$= ei(-\cos\omega t\cos\overline{\omega t - \theta} - \sin\omega t\sin\overline{\omega t - \theta} + k\sin\omega t\cos\overline{\omega t - \theta} - k\sin\overline{\omega t - \theta}\cos\omega t).$$

$$= ei(-\cos\theta + k\sin\theta).$$

Whence, P is also a quaternion (see Z).

$$\text{Scalar part} = -ei\cos\theta = -\text{real power.}$$

$$\text{Vector part} = kei\sin\theta = \text{imaginary power.}$$

$$\text{Tensor part} = ei = TEI.$$

$$\text{Versor part} = -\cos\theta + k\sin\theta.$$

The imaginary power is at right angles to the real power and is not what we call the wattless power. The latter is $ei(1-\cos\theta)$ while the former is $ei\sqrt{(1-\cos^2\theta)}$.

UNIVERSITY OF NEBRASKA.

A CONSTRUCTION FOR DIRECT-READING SCALES FOR THE SLIDE WIRE BRIDGE.

BY EDGAR BUCKINGHAM.

WHEN a large number of rough measurements are to be made on the slide wire Wheatstone's bridge, it is very convenient to have the scale so divided as to read, directly, the ratio of the two segments of the wire, thus obviating the necessity of working out the ratio $a/(1{,}000 - a)$ or looking it up in Obach's table. Such scales are, of course, in common use, but until a few days ago I have never considered how they might be made by a simple geometrical construction. The following method will accomplish the

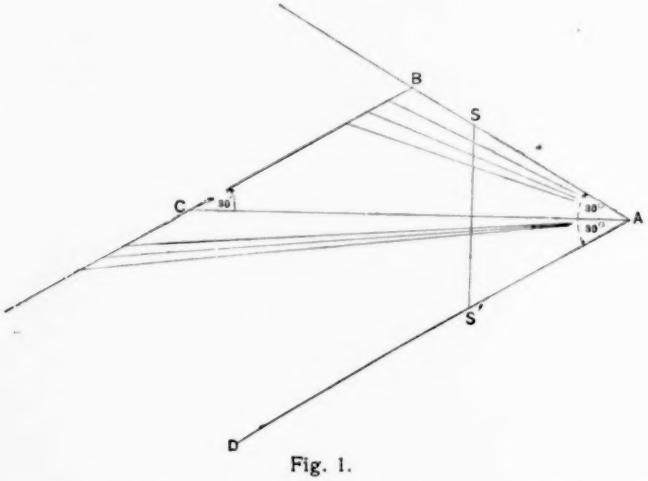

Fig. 1.

desired result; it may very possibly be familiar, but I do not remember seeing it given.

Suppose we have at our disposal some scale of equal parts, such as a meter stick. Construct an isosceles triangle ABC having the equal sides BA and BC each 1 m. long, and the equal angles A and C each 30°. Let the side BC and its continuation beyond

C, be laid off in equal parts (the divisions of the meter stick, for example). Lines drawn from the points thus obtained to the vertex A, will divide in the desired manner any line SS' perpendicular to AC and limited by AB or its continuation, and a line AD drawn symmetrically with AB at an angle of 30° on the other side of AC. Any scale which it is desired to divide may be made to occupy the position SS', and may then be divided at once by the use of a straight edge and a pin at A.

If, for example, the side BC be divided into one hundred equal parts, the successive divisions of SS', beginning at S and numbering them 0.01, 0.02, 0.03, etc., will give the ratio of the length of that part of the scale on the side toward S to the length of the other part on the side toward S'.

BUREAU OF SOILS, WASHINGTON, D. C.,
July 14, 1903.

A METHOD FOR THE COMPARISON OF LOW RESISTANCES.

BY FRANK WENNER.

THE Wheatstone's bridge furnishes our best means for the comparison of resistances but with low resistances certain well-known difficulties arise. Ways have been devised for overcoming these difficulties yet it is believed that the following method has advantages not possessed by the others so it is presented with the hope that it will be of service to some one.

Referring to the diagram let AA' and BB' be the conductors whose resistances are to be compared. These are joined by conductors AB and $A'B'$. Two points X and Y on BB' are connected to the terminals of the battery. The galvanometer is connected to a point Z between X and Y and a point W on AA' such that the bridge is in balance. Move the connections X, Y and Z in

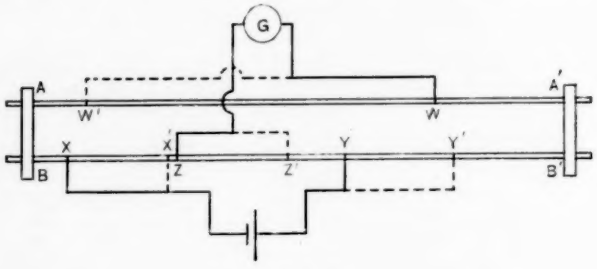

Fig. 1.

the same direction through equal resistances to points X', Y' and Z'. To again balance the bridge the connection W must be moved to a point W' such that the resistance of AA' between W and W' is equal to the resistance of BB' between X and X', Z and Z' or Y and Y'.

The statement hardly needs a proof for it is easily seen that the resistance in each of the four arms of the bridge is increased and decreased by the same amount.

DEPARTMENT OF PHYSICS AND ELECTRICAL ENGINEERING,
IOWA STATE COLLEGE, August, 1903.